Algorithmical and Statistical Modeling, Fall 2012, Exercise Sheet 3 (lecture)

Please return Thursday Oct 11 in class

Problem 1. In the lecture notes, the concepts of an *invariant distribution* (Eq. 4.16) and of *detailed balance* (Eq. 4.17) are given in a format for continuous distributions on $\mathbb{R}^n$ with a pdf.

(a, 25 pts) Please give analog definitions of these two concepts for the (simpler) case of discrete distributions on a finite alphabet $A = \{a_1, ..., a_m\}$, where the transition kernel is given by a stochastic matrix **M**, and a distribution is simply a probability vector **p** of size m. Your definitions should "mimic" the definitions from Eq. 4.16 and Eq. 4.17.

(b, 25 pts) Give alternative (but equivalent) definitions which are expressed in purely algebraic properties of the transition kernel **M** and the distribution vectors **p**.

Problem 2 (25 pts). Consider a "quadratic" alphabet $A^2 = A \times A$ made from symbol pairs over $A = \{a_1, ..., a_m\}$, and a distribution P on $A \times A$. Construct a Gibbs-type Markov chain ($\mathbf{M}$, $\mathbf{p}_0$) sampler [where **M** is a stochastic matrix and $\mathbf{p}_0$ the initial distribution] on this state space. **M** is now of size $m^2 \times m^2$. Describe in formal terms how this Gibbs sampler is constructed.

Problem 3 (25 pts). Construct an concrete example of such a "quadratic alphabet" distribution P, where a Gibbs-type Markov chain ($\mathbf{M}$, $\mathbf{p}_0$) sampler as in Problem 2 would not work (following the inspirations of Figure 4.3).