Exercises Algorithmical & Statistical Modelling, Fall 2012, Sheet 5

Please return on Thu, Nov 22, in the lecture

Problem 1. Prove the equation (5.13), $F(T) = -T \log(Z(T))$, from the lecture notes. Note: this is a mechanical transformation, starting from the definition of F.

Problem 2. A study in modelling cloud patterns. Assume that you are a theoretical physicist familiar with spin glasses, and that you wish to model the stochastic shapes of clouds in a cube of $N \times N \times N$ cubic volume elements (voxels) of sky. Each voxel may be blue or white. Specify microstates and design an energy function that could serve as a basis for a spin-glass model of cloud shapes. Your energy function $E(s)$ should qualitatively account for the follwing meteorological-geometrical facts: (i) The chance of seeing clouds grows with the average relative moisture of the air in your $N \times N \times N$ volume. (ii) Each cloud (i.e., white) voxel represents a condensation of moisture; that is, each white voxel subtracts a constant amount of moisture from the totally available moisture in the volume of sky. (iii) The condensation of a blue air voxel into a white cloud voxel has a tendency to "catalyse" similar changes in adjacent voxels. Build (i), (ii), and (iii) into an energy function (explain the various terms and constants in your function in words). – If you are familiar with the real physics of cloud formation, you will find this picture highly simplified. If you wish, develop your own (better informed, but also more complex) model of cloud shaping and cast it into a spin-glass model. Deliverable: the energy function with its terms explained in plain English.

Bonus task: If you implement a sampling simulation for your cloud formation model, run it and generate a video from it (or several, with different sampling temperatures T or different settings of total available moisture or other global parameters that you invent), and send the video to me together with the simulation code, up to 5 bonus points are awarded. (Bonus points will be added to course point score undiluted at the time of final grade calculation)