Biomedical Signal Processing

Data Engineering Program, spring 2018

Exercise Sheet 2

April 3, 2018

Submission until April 15th, 24.00 to f.hadaeghi@jacobs-university.de

Exercise 1) Discrete Random Processes

1-1. Let $x[n]$ and $y[n]$ be stationary, uncorrelated random signals. Show if $w[n] = x[n] + y[n]$, then $m_w = m_x + m_y$ and $\sigma_w^2 = \sigma_x^2 + \sigma_y^2$.

1-2. Let $e[n]$ denote a white noise sequence and $s[n]$ denote a sequence that is uncorrelated with $e[n]$. Show that the sequence, $y[n] = s[n]e[n]$ is white (that is $\mathcal{E}\{y[n]y[n+m]\} = A\delta[m]$ where A is a constant).

1-3. consider a random signal $x[n] = s[n] + e[n]$ where both $s[n]$ and $e[n]$ are independent, stationary random signals with autocorrelation functions $\varphi_{ss}[m]$ and $\varphi_{ee}[m]$, respectively.

a) Determine expressions for $\varphi_{xx}[m]$ and $\Phi_{xx}(e^{j\omega})$.

b) Determine expressions for $\varphi_{xs}[m]$ and $\Phi_{xs}(e^{j\omega})$.

c) Determine expressions for $\varphi_{xe}[m]$ and $\Phi_{xe}(e^{j\omega})$.

1-4. Consider a random process $x[n]$ that is the response of the LTI system shown in Figure P1-4. In the figure, w[n] represents a real zero-mean stationary white noise process with $\mathcal{E}\{w^2[n]\} = \sigma_w^2$.

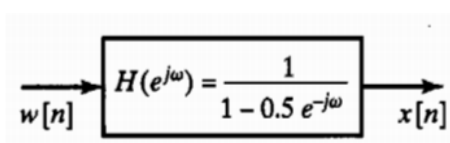

Figure P1-4.

a) Express $\mathcal{E}\{x^2[n]\}$ in terms of $\varphi_{xx}[n]$ and $\Phi_{xx}(e^{j\omega})$.

b) Determine $\Phi_{xx}(e^{j\omega})$, the power density spectrum of $x[n]$.

c) Determine $\varphi_{xx}[n]$, the correlation function of $x[n]$.

Exercise 2) Programming Exercise

For the signals in Table P2, do the following:

a) Generate 2500 samples of the data (i.e., for $n = 0,1, \ldots ,2500$)

b) Use 64-point DFT (in MATLAB) to calculate the Fourier transform of the signal. Plot the amplitude and the phase of the Fourier transform with respect to the frequency, ω.

c) Use 1024-point DFT (in MATLAB) to calculate the Fourier transform of the signal. Plot the amplitude and the phase of the Fourier transform with respect to the frequency, ω. Compare your results with part b.

Table P2.

	Signals
1	$\cos(\frac{n\pi}{2})$
2	$\cos(\frac{n\pi}{2}) + \sin(\frac{n\pi}{3}) + \cos(\frac{3n\pi}{4}) + \sin(\frac{n\pi}{7})$
3	$\frac{dx(t)}{dt} = 0.2\frac{x(t-17)}{1+x(t-17)^{10}} - 0.1x(t)$ *Hint: by means of Euler discretization method, convert the continuous equation to a recurrent difference equation. Then, generate the $x[n]$.* *Here are two samples code:* *1- https://de.mathworks.com/matlabcentral/fileexchange/24390-mackey-glass-time-series-generator* *2- http://lab.fs.uni-lj.si/lasin/wp/IMIT_files/neural/nn05_narnet/#1* Although this signal is produced by a deterministic source, it is known to be a chaotic signal. Therefore, it would be better to calculate the Fourier transform for its autocorrelation.
4	White Gaussian noise with zero-mean and unit variance. *Hint: first calculate the autocorrelation*