Exercises for Computability and Complexity, Spring 2014, Sheet 2

Please return your solutions in class, in the Friday lecture on Feb 21.

Exercise 1 (a) Are the functions $f(n) = \exp(n)$ and $g(n) = \exp(2n)$ polynomially related? **(b)** What about $f(n) = \exp(n)$ and $g(n) = \exp(n^2)$? Prove your answers.

Exercise 2 Show that $L = \{w \in \{1\}^* \mid |w| \text{ is a power of } 2\} \in \mathbf{TIME}(O(n \log n))$, by describing in words (and maybe sketches of interesting configurations) a TM that does this job.

Exercise 3. If one would admit TMs with countably many states, would this extend the set of TM-computable functions on the integers? In other words, is there a function $f: \mathbb{N} \to \mathbb{N}$ which can be computed by some TM with countably infinitely many states, but not by any ordinary TM? Sketch a proof for your answer.

Challenge problem (***optional***) Let $\Sigma_n = \{1,..., n\}$ and $L_n = \{12...n\}$ (i.e. the language that contains only the word 12...*n*). Prove or disprove: a single-tape TM deciding L_n must have at least n states.