Exercises for Computability and Complexity, Spring 2014, Sheet 3

Please return your solutions in class, in the Friday lecture on March 7.

Exercise 1 Consider the ultra-simple TM M with tape alphabet $\Sigma = \{0, 1, \sqcup, \triangleright\}$ and states $\{s, yes, no\}$ that has the following transition table:

$p \in K$	$\sigma \in \Sigma$	$\delta(q, \sigma)$
s	0	$(yes, 0, -)$
s	1	$(s, 1, \rightarrow)$
s	$\sqcup$	$(no, \sqcup, -)$
s	$\triangleright$	$(s, \triangleright, \rightarrow)$

What is the language $L(M)$ decided by M? Describe that language in plain English. Write a RAM program that decides the same language, in the following sense. Your RAM should compute a string function f: $\Sigma^* \rightarrow \{0,1\}$, such that $f(w) = 1$ iff w is in $L(M)$.

Exercise 2 Show that the function plus2: $\mathbb{N} \rightarrow \mathbb{N}$, plus2 $(n) = n + 2$ is primitive recursive. Note: give a formal proof, a plain English explanation isn't enough here.

Exercise 3 Show that the identity function id: $\mathbb{N} \rightarrow \mathbb{N}$, id$(n) = n$ is primitive recursive.

Exercise 4 Consider the set $\boldsymbol{T}$ of all single-tape TMs with tape alphabet $\Sigma = \{0, 1, \sqcup, \triangleright\}$. Design a coding scheme by which every TM M in $\boldsymbol{T}$ becomes coded by a codeword $<M> \in \{0, 1, \#\}^*$. Describe your coding scheme in formal notation and use it to encode the TM from Exercise 1.

Exercise 5 (OPTIONAL) Show that the function *evensquare*: $\mathbb{N} \rightarrow \mathbb{N}$, defined by *evensquare*$(n) = n$ if n is uneven, else *evensquare*$(n) = n^2$, is primitive recursive. You may assume that *square*: $\mathbb{N} \rightarrow \mathbb{N}$, *square*$(n) = n^2$, is primitive recursive. Hint: you may find it helpful to construct *evensquare* from a number of helper functions which you construct before assembling *evensquare*.