Exercises for Computability and Complexity, Spring 2014, Sheet 5

Please return your solutions in the Friday lecture on March 21.

Exercise 1. Evaluate by call-by-value the following terms as far as possible (if you run into a dead end or loop, stop). Then do the same with call-by-name evaluation.

$K(KI(KI))y$
$K(KI(KIy))\ y$
$S\ K\ (KK(KIy))\ y$

Exercise 2 (medium) Express the following combinators in terms of S, K, and I. That is, give an expression for C in each case which consists solely of S, K, and I. *Remark:* this could be done using the systematic procedure given in the proof of Proposition 7.3. The solutions returned by the automatic procedure are however typically far from optimal – they consist in unnecessarily long and complex expressions. Here however I only want you to play around with combinators and find (shorter) solutions by intuition-guided experimentation, which is probably a faster road to success than the systematic procedure.

(a) $Cxy = y$ *(A solution with 2 instances from {S, K, I} exists)*
(b) $Cxyz = y$ *(A solution with 2 instances from { S, K, I} exists)*
(c) $Cxyz = x$ *(A solution with 4 instances from { S, K, I} exists)*

A bit more challenging (optional)

(d) $C_n x_1 \ldots x_n = x_n$ *(A solution with $n+1$ instances from { S, K, I} exists)*

Even more challenging and optional (not easy):

(e) $C_n^i x_1 \ldots x_n = x_i$ This is analog to the projection operator known from recursive functions.

Exercise 3 Construct the combinator C defined by $Cxy = y$ from S and K and I, by meticulously applying the construction method from the proof of Prop. 7.3. (This is the same C as in Exercise 2a). This is a rather short construction (3 lines), but hairy – it takes a lot of care to really get the construction right according to the method in Prop. 7.3. Document all the detailed steps that this construction requires from you. Please use exactly the notation from Prop. 7.3 – starting with setting $t_2[x,y] = y$.