

Exercises for Computability and Complexity, Spring 2014, Sheet 6

Please return your solutions in the Friday lecture on March 28.

Exercise 1 (it's easy, but don't take it easy) Show that the set of combinators $\{A, B, C\}$, defined by $Axy = yx$, $Bxyz = xy$, $Cx = x$ is **not** combinatorially complete. Be formal and rigorous in your proof (there are rather simple proofs for the claim; the main objective of this exercise is to train writing up rigorous proofs).

Exercise 2 (easy). Show that **and true true = true**. You may use **if true** $s t \rightarrow^* s$ and **and** $\equiv \lambda p q. \text{if } p \text{ } q \text{ false}$.

Exercise 3 (medium) Define three λ -terms **a**, **b**, **c** and another λ -term **L** such that **Laa = Lbb = Lcc = Lba = Lca = Lcb = false**, and **Lab = Lac = Lbc = true**. (You may think of **L** as a "properly less than" ordering of **a**, **b**, **c**). Hint: use some of the λ -terms from the lecture notes (Booleans, list operators) in the makeup of **a**, **b**, **c** and **L**.