

Exercises for Computability and Complexity, Spring 2016, Sheet 1

Please return your solutions in class, in the Thursday lecture on Feb 18.

Note: You may work in teams up to size 2.

Exercise 1 Give a formal definition of a version of TMs that use a 2-dimensional grid of memory cells instead of a 1-dimensional tape. Start with a plain English description of your basic ideas and intuitions of how to make a 2-dim grid useful for computations in the TM spirit, that is, what special grid cell symbols you want to use, how to initialize the grid, how to administer input. Then repeat-adapt definition 3.1 in formal rigor, and also give a formal definition of a configuration. Note: there are many ways how a 2-dim TM can be set up in a reasonable way, so there is not a singular "correct" definition. The purpose of this exercise is to hone your formalization skills.

Exercise 2 Give a transition table for a TM that computes the function $f(n) = 2n$. The TM should have the tape alphabet $\{0, 1, \triangleright, \sqcup\}$ and numbers are coded as binary strings by writing them to base 2.

Exercise 3 (a) Are the functions $f(n) = \exp(n)$ and $g(n) = \exp(2n)$ polynomially related? **(b)** What about $f(n) = \exp(n)$ and $g(n) = \exp(n^2)$? Prove your answers.

Exercise 4 Show that $L = \{w \in \{1\}^* \mid |w| \text{ is a power of } 2\} \in \mathbf{TIME}(O(n \log n))$, by describing in words (and maybe sketches of interesting configurations) a TM (with possibly several tapes) that does this job.