Exercises for Computability and Complexity, Spring 2016, Sheet 2

Please return your solutions in class, in the Thursday lecture on March 3. You may work in teams of two; if you do, please hand in a single solution sheet per team.

Exercise 1. If one would admit TMs with countably many states, would this extend the set of TM-computable functions on the integers? In other words, is there a function $f: \mathbb{N} \to \mathbb{N}$ which can be computed by some TM with countably infinitely many states, but not by any ordinary TM? Sketch a proof for your answer.

Exercise 2. Consider the ultra-simple TM M with tape alphabet $\Sigma = \{0, 1, \sqcup, \triangleright\}$ and states $\{s, yes, no\}$ that has the following transition table:

$p \in K$	$\sigma \in \Sigma$	$\delta(q, \sigma)$
s	0	$(yes, 0, -)$
s	1	$(s, 1, \rightarrow)$
s	$\sqcup$	$(no, \sqcup, -)$
s	$\triangleright$	$(s, \triangleright, \rightarrow)$

What is the language $L(M)$ decided by M? Describe that language in plain English. Write a RAM program that decides the same language, in the following sense. Your RAM should compute a string function $f: \Sigma^* \to \{0,1\}$, such that $f(w) = 1$ iff w is in $L(M)$.

Exercise 3. Show that the function plus2: $\mathbb{N} \to \mathbb{N}$, plus2 $(n) = n + 2$ is primitive recursive. Note: give a formal proof, a plain English explanation isn't enough here.

Exercise 4 (a bit more difficult) Show that the function *evensquare*: $\mathbb{N} \to \mathbb{N}$, defined by *evensquare*$(n) = n$ if n is uneven, else *evensquare*$(n) = n^2$, is primitive recursive. You may assume that *square*: $\mathbb{N} \to \mathbb{N}$, *square*$(n) = n^2$, is primitive recursive. Hint: you may find it helpful to construct *evensquare* from a number of helper functions which you construct before assembling *evensquare*.

Challenge problem (***optional***) Let $\Sigma_n = \{1, \ldots, n\}$ and $L_n = \{12\ldots n\}$ (i.e. the language that contains only the word $12\ldots n$). Prove or disprove: a single-tape TM deciding L_n must have at least n states.