

## Exercises for Computability and Complexity, Spring 2016, Sheet 3

Please return your solutions in the Thursday lecture on March 31.

**Exercise 1** Consider the set  $T$  of all single-tape TMs with tape alphabet  $\Sigma = \{0, 1, \sqcup, \triangleright\}$ . Design a coding scheme by which every TM  $M$  in  $T$  becomes coded by a codeword  $\langle M \rangle \in \{0, 1, \#\}^*$ . Describe your coding scheme in formal notation and use it to encode the ultra-simple TM  $M$  with tape alphabet  $\Sigma = \{0, 1, \sqcup, \triangleright\}$  and states  $\{s, yes, no\}$  that has the following transition table:

$p \in K$	$\sigma \in \Sigma$	$\delta(q, \sigma)$
$s$	$0$	$(yes, 0, -)$
$s$	$1$	$(s, 1, \rightarrow)$
$s$	$\sqcup$	$(no, \sqcup, -)$
$s$	$\triangleright$	$(s, \triangleright, \rightarrow)$

**Exercise 2 (rather easy)** Prove that  $H_2 = \{\langle M \rangle; x \mid Code(\langle M \rangle) \text{ and } Standard(x) \text{ and there exists some } y \text{ with } Standard(y) \text{ such that } M(x) = y\}$  from Proposition 6.3 is undecidable.

**Exercise 3 (medium difficult)** Show that the language

$$L = \{\langle M \rangle \in \{0, 1, \#\}^* \mid M \text{ halts on no input}\}$$

is not recursively enumerable.