

## Exercises for Comp & Comp, Spring 2017, Sheet 11

Please return Thursday May 4 in class.

**Problem 1.** Are the functions  $\exp(n)$  and  $\exp(n^2)$  polynomially related? Prove or disprove.

**Problem 2.** Prove or disprove the following claim:

Let  $R \subseteq \Sigma^* \times \Sigma^*$  be a polynomially decidable relation. Furthermore, assume that  $R$  is *constant balanced*, that is, there exists a constant  $C$  such that  $(x, y) \in R$  implies  $|y| \leq C$ . Let  $L = \{w \mid (w, y) \in R \text{ for some } y\}$ . Then  $L \in \mathbf{P}$ .

**Problem 3.** The *Kleene star* of a language  $L$  is  $L^* = \{x_1 \dots x_n \mid n \geq 0, x_i \in L\}$ .

(a) Show that  $\mathbf{NP}$  is closed under the Kleene star.

(b) Show that  $\mathbf{P}$  is closed under the Kleene star.

*Note.* This is a math flavored problem which will sharpen your analytical thinking powers. (a) and (b) are about equally difficult in my view. When you have thought about the problem for a while, finding a solution is not in fact difficult – very straightforward proofs exist. The proofs that came to my mind are each about 10 lines of formula+text in a fontsize like this here. Give a serious try to at least one of the two claims (this sheet will get you full marks even if you only work on (a) or (b)).