Exercises for Computability and Complexity, Spring 2017, Sheet 8 – Solutions

Please return your solutions in the Thursday lecture on April 6

Exercise 1 (easy). Show that **and true true** $=$ **true**. You may use **if true** $s\ t \to^* s$ and **and** $\equiv$ $\lambda pq.$ **if** $p\ q$ **false.**

Solution.

and true true expands to ($\lambda pq.$ **if** $p\ q$ **false**) **true true**, which results in **if true true false**, which in turn yields **true**.

Exercise 2 (medium) Define three λ-terms **a**, **b**, **c** and another λ-term **L** such that **Laa** = **Lbb** = **Lcc** = **Lba** = **Lca** = **Lcb** = **false**, and **Lab** = **Lac** = **Lbc** = **true**. (You may think of **L** as a "properly less than" ordering of **a**, **b**, **c**). Hint: use some of the λ-terms from the lecture notes (Booleans, list operators) in the makeup of **a**, **b**, **c** and **L**.

Solution. There are many solutions. One brutal possibility is to set **a** ≡ [**true false false**], **b** ≡ [**false true false**], **c** ≡ [**false false true**] and

L ≡ $\lambda xy.$ **or** (**or** (**and** (**first** x) (**second** y))
(**and** (**first** x) (**second** (**second** y))))
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