Exercises for Comp & Comp, Spring 2018, Sheet 10

Please return Thursday April 26 in class.

Problem 1. Prove or disprove the following claim:

Let $R \subseteq \Sigma^* \times \Sigma^*$ be a polynomially decidable relation. Furthermore, assume that R is *constant balanced*, that is, there exists a constant C such that $(x, y) \in R$ implies $|y| \le C$. Let $L = \{w \mid (w, y) \in R \text{ for some } y\}$. Then $L \in \mathbf{P}$.

Problem 2. The *Kleene star* of a language L is $L^* = \{x_1 ... x_n \mid n \ge 0, x_i \in L\}$.

(a) Show that **NP** is closed under the Kleene star.
(b) Show that **P** is closed under the Kleene star.

Note. This is a "math flavored" problem, which will sharpen your analytical thinking powers. (a) and (b) are about equally difficult in my view. When you have thought about the problem for a while, finding a solution is not in fact difficult – very straightforward, natural proofs exist. The proofs that came to my mind are each about 10 lines of formula+text in a fontsize like this here. Give a serious try to at least one of the two claims (this sheet will get you full marks even if you only work on (a) or (b)).