Exercises for Computability and Complexity, Spring 2018, Sheet 2

Please return your solutions in class, in the Thursday lecture on February 22. You may work in teams of 2.

Exercise 1 (a) Are the functions $f(n) = \exp(n)$ and $g(n) = \exp(2n)$ polynomially related? **(b)** What about $f(n) = \exp(n)$ and $g(n) = \exp(n2)$? Prove your answers.

Exercise 2. Consider the ultra-simple TM M with tape alphabet $\Sigma = \{0, 1, \sqcup, \rhd\}$ and states $\{s, yes, no\}$ that has the following transition table:

$p \in K$	$\sigma \in \Sigma$	$\delta(q, \sigma)$
s	0	$(yes, 0, -)$
s	1	$(s, 1, \rightarrow)$
s	$\sqcup$	$(no, \sqcup, -)$
s	$\rhd$	$(s, \rhd, \rightarrow)$

What is the language $L(M)$ decided by M? Describe that language in plain English. Write a RAM program that decides the same language, in the following sense. Your RAM should compute a string function $f: \Sigma^* \rightarrow \{0,1\}$, such that $f(w) = 1$ iff w is in $L(M)$.

Challenge problem (***optional, not easy, has been solved in the past by a few students***) Let $\Sigma_n = \{1, ..., n\}$ and $L_n = \{12...n\}$ (i.e. the language that contains only the word $12...n$). Prove or disprove: a single-tape TM deciding L_n must have at least n states.