

Exercises for Computability and Complexity, Spring 2018, Sheet 5 – Solutions

Please return on Thursday, March 15, in class. As usual you are invited but not requested to work in teams of size at most 2.

Exercise 1 This problem concerns, and uses the notation from, Section 6.4 from the lecture notes (undecidability of FOL). Show that the language

$$L_{\rightarrow} = \{ \langle \xi \rightarrow \zeta \rangle \in \{0,1,\#\}^* \mid \xi, \zeta \text{ are } S\text{-expressions, and } \xi \models \zeta \}$$

is undecidable.

Solution. Let φ be an S -expression. Then

$$\begin{aligned} \varphi \in L_{\text{FOL}} & \quad \Leftrightarrow \quad \varphi \text{ is a tautology} \\ & \quad \Leftrightarrow \quad \varphi \models t \equiv t \quad (\text{where } t \text{ is some } S\text{-term}) \\ & \quad \Leftrightarrow \quad \langle \varphi \rightarrow t \equiv t \rangle \in L_{\rightarrow} \end{aligned}$$

and hence we could decide L_{FOL} if we could decide L_{\rightarrow} , contradiction.