## Exercises for Computability and Complexity, Spring 2018, Sheet 7 – Solutions

*Please return your solutions in the Thursday lecture on April 5*

**Exercise 1 (easy)**. Show that **and true true** = **true**. You may use **if true** $s\ t \to^* s$ and **and** ≡ $\lambda pq.$ **if** $p\ q$ **false.**

**Solution.**

**and true true** expands to ($\lambda pq.$ **if** $p\ q$ **false**) **true true**, which results in **if true true false**, which in turn yields **true**.

**Exercise 2 (medium)** Define three λ-terms **a**, **b**, **c** and another λ-term **L** such that **Laa** = **Lbb** = **Lcc** = **Lba** = **Lca** = **Lcb** = **false**, and **Lab** = **Lac** = **Lbc** = **true**. (You may think of **L** as a "properly less than" ordering of **a**, **b**, **c**). Hint: use some of the λ-terms from the lecture notes (Booleans, list operators) in the makeup of **a**, **b**, **c** and **L**.

**Solution.** There are many solutions. One brutal possibility is to set **a** ≡ [**true false false**], **b** ≡ [**false true false**], **c** ≡ [**false false true**] and

**L** ≡ $\lambda xy.$ **or** (**or** (**and** (**first** $x$) (**second** $y$))
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