Exercises for FFL, Fall 2017, sheet 10 - solutions

Return Thursday Dec 7, in class.

Exercise 1. Here are some plain English sentences. They cannot be coded in FOL in natural ways, for a diversity of reasons (but they can be coded in other logical systems that are used in mathematics, AI and linguistics). For each of the sentences below, give a brief statement why you think formalizing the sentence in FOL cannot be properly done.

1. I am a human.
2. Peter loves everything about Anne.
3. Is this a car?
4. There exist infinitely many stars.

Solution.

1. There is no way in FOL to express "I". The following is NOT a proper formalization. Assuming that I am the only Jacobs professor teaching FOL, attempts like

   $\exists^{=1} x$ (JacobsProf x $\wedge$ (teaches-FOL $x \wedge$ human x))

   fail to express the important function of the word "I" that the statement made ("I am a human") is made *by* the unique person $\exists^{=1} x$ (JacobsProf x $\wedge$ teaches-FOL x …)

2. A more formal way to say the same is "Peter loves all the properties that Anne has". This is a second-order logic statement and could be expressed as

   $\forall P$ (P Anne $\to$ ($\forall x$ $Px \to$ loves Peter x))

3. FOL cannot code questions. The only sentences that can be expressed in FOL are claims of facts ("factual statements"). Similarly, FOL can't code commands, exclamations of surprise, or any other sentences that carry with them a function in discourse that is not just a factual statement. In order to express such other *modes* of statements, one can (in fact, must) use so-called *modal logics*.

4. FOL cannot express "there exist infinitely many…". If one wants to express that, one has several options. One option is to add a new quantifier $\exists^{\text{inf}}$, thereby extending the syntax and semantics of FOL. Observing that it is easy to express "there are more than n things of…" for any natural number n in FOL, that is, using $\exists^{>n}$ is possible in FOL, one can also capture infinity by admitting infinitely long conjuctions of *S*-expressions, here:

   $\exists^{>1} x$ star $x \wedge \exists^{>2} x$ star $x \wedge \exists^{>3} x$ star $x \wedge$ ...

   This leads to an extension of FOL.

Exercise 2. Give a formal *semantic* proof (proof of soundness as done in the LN for some of the basic rules of the sequent calculus) of the *chain* rule

$\Gamma \quad \psi$
$\Gamma \quad \psi \quad \varphi$

$\Gamma \quad \varphi$

Solution. Let $\mathcal{I}$ be a model of Γ. Because of the first premise $\Gamma \vDash \psi$, $\mathcal{I}$ is also a model of ψ, hence $\mathcal{I}$ is a model of Γ and ψ, hence (using the second premise), $\mathcal{I}$ is a model of φ. We have thus shown that every model $\mathcal{I}$ of Γ is also a model of φ, which is the conclusion of that rule.

Exercise 3. Give a derivation of the following rule:

$(\varphi \vee \neg \varphi)$

in the sequent calculus!

Solution:

1.	φ	φ	(Pre)
2.	φ	$(\varphi \vee \neg \varphi)$	(∨ Con a.) on 1.
3.	$\neg \varphi$	$\neg \varphi$	(Pre)
4.	$\neg \varphi$	$(\varphi \vee \neg \varphi)$	(∨ Con b.) on 3.
5.		$(\varphi \vee \neg \varphi)$	(Cas) on 2. and 4.