Exercises for FLL, Fall 2017, sheet 1 – Solutions

Return Thursday Sep 28, in class

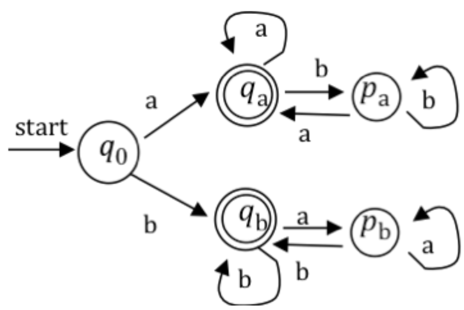
Note: you may work in teams of 2 if you wish. If you do, hand in a single solution sheet for both of you.

Exercise 1 (a) How many words exist over the alphabet Σ = {1}? and over the alphabet Σ = {a, b}? **(b)** How many words of length *n* exist over an alphabet of size *k*? **(c)** How many languages exist over the alphabets from (a) and (b)? **(d)** How many languages of words of length *n* exist over an alphabet of size *k*? **(e, a bit more difficult, optional)** Show that there are countably infinite many *finite* languages over Σ = {a, b}? *Hint: show two things. First, that there are **at least** as many finite languages as there are natural numbers – show this by giving an injective map from ℕ to the set of finite languages. Second, show that there are **at most** as many finite languages as there are natural numbers – show this by giving an injective map from the set of finite languages to ℕ.*

Solution: (a) The words over *S* = {1} are ε, 1, 11, 111, ... – that is, as many as there are integers, that is, countably infinite many. The words over *S* = {a, b} can be listed in a sequence, shortest first, sorted alphebetically for same size: ε, a, b, aa, ab, ba, bb, aaa, aab, ... – that is, again countably many. **(b)** k^n many. **(c)** In both cases, |Σ*| = | ℕ |, that is, there are are | $2^{\mathbb{N}}$ | = | ℝ | many languages over these alphabets – indeed, over any finite alphabet there are | ℝ | many languages. **(d)** Since there are k^n many words of length *n* over a symbol set of size *k*, there are $2^{(k^n)}$ many such languages. **(e)** Let *F* be the set of finite languages over {a, b}. (I) "***at least** as many finite languages as there are natural numbers":* there are many ways to define an injection β: ℕ → *F*. For instance, $\beta(n) := \{a, a^2, ..., a^n\}$ does it. (II) "***at most** as many finite languages as there are natural numbers":* It is clear that a finite language over over Σ = {a, b} can be written as a word over the over Σ' = {a, b, _}. Example: *L* = {a, aa, ab, bbba} can be written as a_aa_ab_bbba. We know that there are | ℕ | many words over Σ' = {a, b, _}, and each finite language over Σ corresponds to one of these words, so there are no more finite languages over Σ than words over Σ', that is no more than | ℕ | many.

Exercise 2. Design a DFA which accepts the language *L* = { *w* ∈ {*a*, *b*}* | |*w*| > 0, *and the last symbol in w is equal to the first*}. Describe your DFA both by a complete transition table and through a graphical transition diagram.

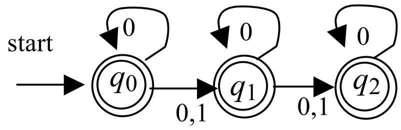
Solution. The simplest DFA which does this has 5 states:



Transition table:

		a	*b*
→	q_0	q_a	q_b
	q_a*	q_a	p_a
	q_b*	p_b	q_b
	p_a	q_a	p_b
	p_b	p_b	q_b

Exercise 3. Describe the language accepted by the NFA shown below in plain English.



Solution. This NFA accepts all words over the binary alphabet which contain at most two 1's.

Exercise 4. Construct a DFA equivalent to the NFA depicted above, *using the subset construction*. Present your DFA by a transition diagram.

Solution.