Exercises for FFL, Fall 2017, sheet 8

Return Thursday Nov 23, in class.

Exercise 1. Let $S = \{c, f\}$ be a signature consisting of a constant symbol c and a binary function symbol f. Let $\mathcal{A} = (A, c^A, f^A)$ be the S-structure over carrier $A = \{1, 2\}$ with $c^A = 1$ and f^A given by $f^A(1, 1) = 1, f^A(1, 2) = 2, f^A(2, 1) = f^A(2, 2) = 1$. Your task: represent $\mathcal{A}$ as a set, coding lists by iterated nested pairs (right-associative) and representing these (nested) pairs as sets (so ultimately there will be only "{}" symbols, no more "()" symbols). Using a text processor's copy-paste capabilities saves a lot of time in this task, prevents copy errors, and makes it possible for the TAs to read your result, so please use a text processor for this task.

Exercise 2.
Let $S = \{<\}$, where $<$ is a binary relation symbol. Characterize in words the class of all S-structures $\mathcal{A}$ which are models of

$$\varphi = \forall x_1 \forall x_2 \forall x_3(((((\neg\, x_1 = x_2 \wedge \neg\, x_2 = x_3) \wedge \neg\, x_1 = x_3) \wedge < x_1\, x_2) \wedge < x_2\, x_3) \rightarrow \neg < x_3\, x_1).$$

Furthermore, give two concrete S-structures, one of which is a model of φ and the other isn't. Present your structures by specifying the carrier and the extension of $<^A$.

Exercise 3. Let S = {father-of, Tom, Anne}, where father-of is a unary function symbol and Tom, Anne are constant symbols. Give a concrete S-structure which is *not* a model of the S-expression "father-of Anne = Tom". And give a concrete S-structure which *is* a model of the S-expression "father-of Tom = Anne".

Exercise 4. You know that a binary relation is called an *equivalence relation* if it is (i) reflexive, (ii) symmetric, and (iii) transitive. Let $S = \{\equiv\}$, where $\equiv$ is a binary relation symbol. Give an S-expression φ such that in any S-structure $\mathcal{A} = (A, \equiv^{\mathcal{A}})$, which is a model of φ, $\equiv^{\mathcal{A}}$ is an equivalence relation on A.