Exercises for FFL, Fall 2017, sheet 9 - solutions

Return Thursday Nov 30, in class.

Exercise 1. Here we go… again. Formalize in FOL the following English statements. In each case, declare the type of each signature symbol that you use. Use the strict syntax declared by the definitions in the lecture notes – not using more or less brackets than demanded by those strict syntax rules.

1. When a glass of water is half full, it is also half empty and vice versa.
2. Obama is the President of the United States.
3. Some dogs are stupid.
4. A dog can't be clever and stupid at the same time, but some humans are.
5. A knife lies on my breakfast table.
6. Barber's coffee bar is located at the south corner of the intersection of High Street and Barber's street.

Solution. Note: the following are suggestions only; formalizing natural language statements can always be done in many different ways. For brevity I annotate the type of a symbol directly in the S-expression by an upper index, e.g. father_of ^{1f} means that father_of is a unary function symbol.

1. $\forall x\, (\text{isGlassOfWater}^{1R}\, x \rightarrow (\text{halfFull}^{1R}\, x \leftrightarrow \text{halfEmpty}^{1R}\, x))$
2. $\text{PresidentOf}^{\,1f}\ \text{US}^{\,c} = \text{Obama}^{\,c}$
3. $\exists\ x\ (\text{isDog}^{1R}\, x \wedge \text{stupid}^{1R}\, x)$
4. $\forall x\, (\text{isDog}^{1R}\, x \rightarrow\ (\text{stupid}^{\,1R} x \leftrightarrow \neg\ \text{clever}^{1R}\, x)) \wedge \exists\ x\ (\text{human}^{1R}\, x \wedge \text{stupid}^{1R}\, x \wedge \text{clever}^{1R}\, x)$
5. $\exists\ x\ (\text{knife}^{1R}\, x \wedge \text{lies-on}^{2R}\, x\ \text{myBreakfastTable}^{\,c})$
6. $\text{located-at}^{2R}\ \text{BarbersBar}^{\,c}\ \text{Southcorner-of}^{\,1f}\,\text{Intersection-of}^{1f}\ \text{HighStreet}^{\,c}\ \text{BarberStreet}^{\,c}$

Exercise 2. Consider the following propositions which express that the binary relation R is an equivalence relation:

φ_1: $\forall x\, Rxx$ φ_2: $\forall x \forall y\, (Rxy \rightarrow Ryx)$ φ_3: $\forall x \forall y \forall z\, ((Rxy \wedge Ryz) \rightarrow Rxz)$

Show that none of these propositions is entailed by the others by presenting $\{R\}$-structures that are models of two of the propositions, but not of the third.

Solution. Here is one possibility.
(i) A model of φ_2 and φ_3 but not of φ_1: $\mathcal{A} = (\{1\}, \varnothing)$
(ii) A model of φ_1 and φ_3 but not of φ_2: $\mathcal{A} = (\{1,2\}, \{\{1,1\},\{1,2\},\{2,2\}\})$
(iii) A model of φ_1 and φ_2 but not of φ_3:
$\mathcal{A} = (\{1,2,3\}, \{\{1,1\},\{2,2\},\{3,3\},\{1,2\},\{2,1\},\{2,3\},\{3,2\}\})$

Exercise 3. Using the rules from the sequent calculus, derive the rule

$$\frac{\qquad}{\Gamma\ \neg(\varphi \vee \neg\varphi)\ \zeta} \qquad \text{(for any } \Gamma, \varphi, \zeta)$$

Below I give the first 5 steps of a derivation that finishes after the 8th step. You can use this beginning.

(1) $\Gamma\ \neg(\varphi \vee \neg\varphi)$	$\neg(\varphi \vee \neg\varphi)$	(Pre)
(2) $\Gamma\ \varphi$	φ	(Pre)
(3) $\Gamma\ \neg\varphi$	$\neg\varphi$	(Pre)
(4) $\Gamma\ \varphi$	$(\varphi \vee \neg\varphi)$	($\vee$Con,a) on (2)
(5) $\Gamma\ \neg\varphi$	$(\varphi \vee \neg\varphi)$	($\vee$Con,b) on (3)
(6) Γ	$(\varphi \vee \neg\varphi)$	(Cas) on (4),(5)
(7) $\Gamma\ \neg(\varphi \vee \neg\varphi)$	$(\varphi \vee \neg\varphi)$	(Ant) on (6)
(8) $\Gamma\ \neg(\varphi \vee \neg\varphi)$	ζ	(Con) on (1),(7)