

Exercises for FFL, Fall 2014, sheet 10

Return Wednesday Nov 19, in class.

Exercise 1. Let $S = \{<\}$, where $<$ is a binary relation symbol. Characterize in words the class of all S -structures \mathcal{A} which are models of

$$\varphi = \forall x_1 \forall x_2 \forall x_3 (((\neg x_1 = x_2 \wedge \neg x_2 = x_3) \wedge \neg x_1 = x_3) \wedge < x_1 x_2 \wedge < x_2 x_3) \rightarrow \neg < x_3 x_1)$$

and give two concrete S -structures, one of which is a model of φ and the other isn't. Present your structures (i) in an intuitive graph-like representation, (ii) formally as sets. How many non-isomorphic models does φ have?

Exercise 2. Consider the following propositions which express that the binary relation R is an equivalence relation:

$$\varphi_1 = \forall x Rxx$$

$$\varphi_2 = \forall x \forall y (Rxy \rightarrow Ryx)$$

$$\varphi_3 = \forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz)$$

Show that none of these propositions is entailed by the others by presenting $\{R\}$ -structures that are models of two of the propositions, but not of the third.