Exercises for FLL, Fall 2015, sheet 3

Return Tuesday Sep 29, in the lecture

Exercise 1. Prove or disprove the following conjecture:

Let M be some regular language over $\Sigma = \{0,1\}$. Define $L_{|M|} = \{0^n \in \{0\}^* \mid n = |v|$ for some word $v \in M\}$. Then L is regular.

Note: this is easy to solve using the tool of language homomorphisms. We skipped them in class. If you wish, read up on them (Def. 3.15, Prop 3.11 in LNs) and use them for this problem. It is also possible to solve this problem without homomorphisms, also not difficult.

Exercise 2. Prove or disprove the following conjecture:

Let M, N be regular languages over $\Sigma = \{0,1\}$. Define $L = \bigcup_{k=|v| \text{ for some } v \in M} N^k$. Then L is regular.

Exercise 3 A *sequence language* L over Σ is a language with two properties: (i) for each $n \geq 0$, there exists exactly one word in L of that length; (ii) if $u, v \in L$, $|u| < |v|$, then u is a prefix (= initial subword) of v.

a. Give an example of a regular sequence language.

Prove that every regular sequence language L is ultimately cyclic, that is, there exist words w and v such that L is the set of all initial substrings of the infinite sequence *wvvvv...* . *Hint*: you will benefit from the PL here.

Note. This was a midterm question in a long-time-ago FLL course.