Exercises for FFL, Fall 2015, sheet 8

Return Tuesday Nov 10, in class.

Exercise 1. Formalize in FOL the following English statements. In each case, declare the type of each signature symbol that you use. Use the strict syntax declared by the definitions in the lecture notes – not using more or less brackets than demanded by those strict syntax rules.

1. My father is an airline pilot.
2. Nobody is perfect.
3. Men and women have the same rights.
4. New York is further away from Bremen than Hannover.

Solution. Note: the following are suggestions only; formalizing natural language statements can always be done in many different ways.

1. Airline-Pilot Father-of I *where* Airline-Pilot is unary pred. symbol, Father-of unary function symbol, I constant symbol
2. $\forall x$ (human $x \rightarrow \neg$ perfect x) *where* human and perfect are unary pred. symbols
3. $\forall x_1 \forall x_2 \forall x_3$ (((man $x_1 \wedge$ woman x_2) $\wedge$ right x_3) $\rightarrow$ (has x_1 $x_3 \leftrightarrow$ has x_2 x_3)) *where* man, woman, right are unary predicate symbols, has is a binary relation symbol.
4. Greater-than dist NY Bremen dist Hannover Bremen *where* Greater-than is binary relation symbol, dist is binary function symbol, NY, Bremen, Hannover are constant symbols.

Exercise 2. Let $S = \{<\}$, where $<$ is a binary relation symbol. Characterize in words the class of all S-structures $\mathcal{A}$ which are models of

$$\varphi = \forall x_1 \forall x_2 \forall x_3(((((\neg\, x_1 = x_2 \;\wedge\; \neg\, x_2 = x_3) \wedge \neg\, x_1 = x_3) \wedge < x_1\, x_2) \wedge < x_2\, x_3) \rightarrow \neg < x_3\, x_1)$$

and give two concrete S-structures, one of which is a model of φ and the other isn't. Present your structures (i) in an intuitive graph-like representation, (ii) formally as sets.

Solution. The models of φ are exactly those $\{<\}$-structures that contain no $<$-cycle of length 3. The simplest S-structure that is a model of φ is given by a singleton set A and empty $<$, that is, $A = \{a\}$ and $<^A = \emptyset$ (graph-like representation: a single point). The simplest S-structure that is not a model of φ is an isolated 3-cycle of $<$, that is, $A = \{a, b, c\}$ and $<^A = \{(a,b), (b,c), (c,a)\}$: