Exercises for FFL, Fall 2016, sheet 9

Return Thursday Nov 10, in class.

Exercise 1.
Let $S = \{<\}$, where $<$ is a binary relation symbol. Characterize in words the class of all S-structures $\mathcal{A}$ which are models of

$$\varphi = \forall x_1 \forall x_2 \forall x_3 (((((\neg\, x_1 = x_2 \;\wedge\; \neg\, x_2 = x_3) \wedge \neg\, x_1 = x_3) \wedge < x_1\, x_2) \wedge < x_2\, x_3) \rightarrow \neg < x_3\, x_1)$$

and give two concrete S-structures, one of which is a model of φ and the other isn't. Present your structures (i) in an intuitive graph-like representation, (ii) formally as sets.

Exercise 2. Let S = {father-of, Tom, Anne}, where father-of is a unary function symbol and Tom, Anne are constant symboles. Give a concrete S-structure which is *not* a model of the S-expression "father-of Anne = Tom". And give a concrete S-structure which *is* a model of the S-expression "father-of Tom = Anne".

Exercise 3. You know that a binary relation is called an *equivalence relation* if it is (i) reflexive, (ii) symmetric, and (iii) transitive. Let $S = \{\equiv\}$, where $\equiv$ is a binary relation symbol. Give an S-expression φ such that in any S-structure $\mathcal{A} = (A, \equiv^{\mathcal{A}})$, which is a model of φ, $\equiv^{\mathcal{A}}$ is an equivalence relation on A.