Miniquiz preparation sheet for FFL, Fall 2016

This is an extra sheet with solved problems of miniquiz preparation. Not a homework. Use this sheet in whatever way you find wise.

Problem 1 (was a miniquiz problem some years ago). Here is the "rule of the excluded middle" (Latin, for the educated: "tertium non datur"):

$$\frac{}{(\varphi \vee \neg\varphi)} \qquad \text{(for all } \varphi\text{)}$$

Give a derivation of this rule in the sequent calculus. Hint: the first two rules to apply are (Pre) and (∨Con)(a), and with these two rules plus (∨Con)(b) and (Cas) one can build a derivation with 5 steps. (Solution at end of this sheet).

Problem 2. Using the rules from the sequent calculus, derive the rule

$$\frac{}{\Gamma\ \neg(\varphi \vee \neg\varphi)\ \zeta} \qquad \text{(for any } \Gamma, \varphi, \zeta\text{)}$$

Remark: Not an easy one if you are not used to this kind of problem. Don't give up too early. I found a derivation in 8 steps.

Solution to problem 1 (one among many possibilities)

1.	φ	φ	(Pre)
2.	φ	$(\varphi \vee \neg\varphi)$	(∨Con)(a) on 1.
3.	$\neg\varphi$	$\neg\varphi$	(Pre)
4.	$\neg\varphi$	$(\varphi \vee \neg\varphi)$	(∨Con)(b) on 3.
5.	$(\varphi \vee \neg\varphi)$		(Cas) on 2., 4.

Solution to problem 2. One possible derivation is:

(1)	$\Gamma\ \neg(\varphi \vee \neg\varphi)$	$\neg(\varphi \vee \neg\varphi)$	(Pre)
(2)	$\Gamma\ \varphi$	φ	(Pre)
(3)	$\Gamma\ \neg\varphi$	$\neg\varphi$	(Pre)
(4)	$\Gamma\ \varphi$	$(\varphi \vee \neg\varphi)$	(∨Con,a) on (2)
(5)	$\Gamma\ \neg\varphi$	$(\varphi \vee \neg\varphi)$	(∨Con,b) on (3)
(6)	Γ	$(\varphi \vee \neg\varphi)$	(Cas) on (4),(5)
(7)	$\Gamma\ \neg(\varphi \vee \neg\varphi)$	$(\varphi \vee \neg\varphi)$	(Ant) on (6)
(8)	$\Gamma\ \neg(\varphi \vee \neg\varphi)$	ζ	(Con) on (1),(7)