

## GenICT Fall 2015, Module 2 (Boolean logic), HW 2

Please upload a pdf file with your solutions (legibly hand-written and scanned, or typeset documents) to <https://jgrader.de> by Tuesday, Oct 13, 23:59.

**Problem 1 (20 pts).** Consider the following truth table:

$X$	$Y$	$Z$	$\varphi$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Give a Boolean expression for  $\varphi$  in CNF!

**Problem 2 (30 pts).** Consider the following facts (well, facts...).

- (a) Everybody who has a good sense of hearing can sing well.
- (b) Nobody is a true musician who cannot inspire her audience.
- (c) Nobody without a good sense of hearing can inspire her audience.
- (d) Nobody except a true musician can write a symphony.

Question: which characteristics must everybody necessarily have who wrote a symphony? First formalize each of the facts (a) – (d) as an implication between two propositions, which gives you four implications, and then use those to find out the answer to the question.

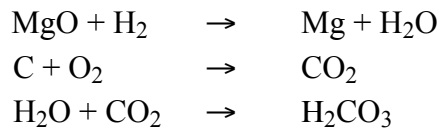
Here is an example of how an English sentence similar to the sentences (a)-(d) above can be cast as an implication:

- (x) No human is immortal.

This is turned into an implication by introducing a Boolean variable  $H$  to be understood as "is human" and a Boolean variable  $M$  meaning "is mortal", then (x) becomes formalized as  $H \rightarrow \neg M$ .

For your formalization of (a) – (d), use the Boolean variables  $H$  = has a good sense of hearing,  $S$  = can sing well,  $T$  = is true musician,  $I$  = can inspire audience,  $W$  = can write symphony.

**Problem 3 (30 pts).** Another exercise in formalizing a bit of the real world. Assume you have chemical apparatuses at your disposal to synthesize



Assume further that you have the raw chemicals MgO, H<sub>2</sub>, O<sub>2</sub> and C available. Find a Boolean formula  $\varphi$  whose *unsatisfiability* is equivalent to synthesizability of H<sub>2</sub>CO<sub>3</sub> in your lab. Hint: for designing  $\varphi$ , associate each chemical substance with a Boolean variable  $X$ , whose intuitive interpretation would be " $X$  is true if the substance that it represents is available or can be synthesized". Then write each of the "synthesizability" rules above as an implication. For instance, use the following Boolean variables for our chemicals: MgO =  $X$ , H<sub>2</sub> =  $Y$ , H<sub>2</sub>O =  $Z$ , C =  $U$ , O<sub>2</sub> =  $V$ , CO<sub>2</sub> =  $W$ , H<sub>2</sub>CO<sub>3</sub> =  $R$ , Mg =  $S$ . The first synthesization rule above would then lead to the two implications  $X \wedge Y \rightarrow S$ ,  $X \wedge Y \rightarrow Z$ .

**Problem 4 (20 pts).** Is  $e^{2n} = O(e^n)$ ? Prove or disprove.

**Solution.**  $e^{2n} = O(e^n)$  is false. Proof by contradiction. Assume it is true. Then there is some constant  $c$  such that  $e^{2n} \leq ce^n$  for  $n$  larger than some  $n_0$ . This is equivalent to  $\frac{e^{2n}}{e^n} = e^n \leq c$  for all  $n$  larger than some  $n_0$ . But for all  $n > \log(c)$  one has  $e^n > c$ , so no such  $c$  can exist.