PSM Fall 2015, Exercise Sheet 3

Return on Monday Sep 28 in class

Note. You are encouraged to work in teams of two – but no larger. If you work in a team, submit only a single sheet with both names indicated on it. Nicely type-set solutions are highly appreciated.

Problem 1 Let $\Omega = \{\omega_1, \ldots, \omega_5\}$ be a (uncommonly small) universe. Define two RVs $X, Y : \Omega \to \{\text{red, blue, green}\}$ which are identically distributed but not identical.

Problem 2 Let $X_1 : \Omega \to \{1, 2, 3, 4, 5, 6\}, X_2 : \Omega \to \{\text{fast, slow}\}$ be two RVs with values in $S_1 = \{1, 2, 3, 4, 5, 6\}$ and $S_2 = \{\text{fast, slow}\}$. Invent and fully specify a joint distribution of these two RVs which makes X_1 independent of X_2. (Note. In the lecture notes posted on Sept 21 Definition 9.1 of independence was too narrowly formulated. I made it more general in an update of the LNs that is now online at the course homepage. You need the more general definition for this problem.)

Problem 3 Let X denote the number of busy servers at the checkout counters of Apetito at 1p.m. The cumulative distribution function of X is

x	0	1	2	3	4
$F(x)$	0.20	0.50	0.80	0.90	1.00

1. Find the probability that at most one server is busy.
2. Find the probability mass function of X.
3. Find the probability that three or more servers are busy.

Problem 4 Let Z denote a random variable with probability mass function

z	-2	-1	0	1	2	4
$p(z)$	0.10	0.30	k	0.05	0.25	0.21

1. What is k?
2. Find the cumulative distribution function of Z.
3. Find $P(-1 \le Z \le 1)$, $P(Z > 0)$, $P(Z \le 2)$, and $P(-2 \le Z < 2)$.

Problem 5 I have an unfair coin for which $P(\text{'Head occurs'}) = p$, where $0 < p < 1$. I toss the coin repeatedly until I observe a heads for the first time. Let Y be the total number of coin tosses that I have to perform.

1. Give a full definition of the random variable Y.
2. Find the probability mass function of Y.
3. Check that the function you have found in subquestion 2 is indeed a probability mass function.
4. For $p = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ compute the probability $P(2 \le Y \le 5)$.

Problem 6 A random variable X has the cumulative distribution function

$$F(x) = \frac{e^x}{1 + e^x}$$

a) Is X a discrete random variable, a continuous random variable, or something else?

b) What is the probability that $0 \le X \le 1$? Simplify your answer as much as possible. (Do NOT leave it as some sort of sum or integral).

c) Sketch the corresponding PDF and mark the following probabilities in the graph: $P(0 \le X \le 1)$, $P(X \ge 4)$, and $P(X \le 2)$.