Principles of Statistical Modeling

Some distributions

Uniform distribution on interval [c,d]

Definition 7 *The distribution of the random variable* $X : \Omega \to [c, d]$ *having the following PDF is called* ***Uniform Distribution on*** $[c, d]$

$$f(x) = \begin{cases} \frac{1}{d-c}, & \text{for } c \leq x \leq d \\ 0, & \text{otherwise.} \end{cases}$$

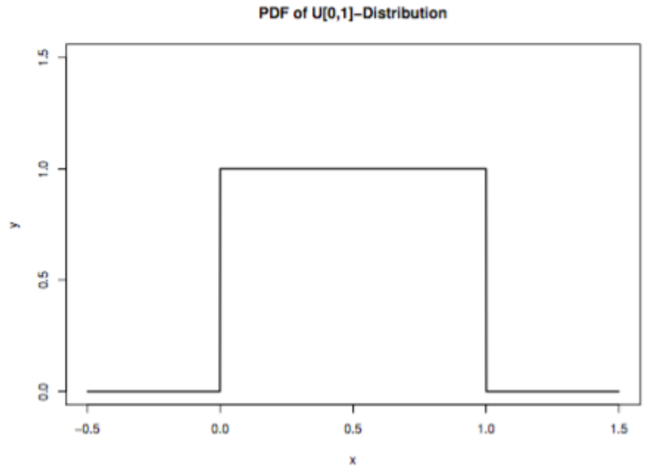
The CDF of a uniformly distributed RV X is given by

$$F(x) = \begin{cases} 0 & \text{for } x < c \\ \frac{x-c}{d-c}, & \text{for } c \leq x \leq d \\ 1, & \text{for } x > d. \end{cases}$$

Uniform distribution on [0,1]

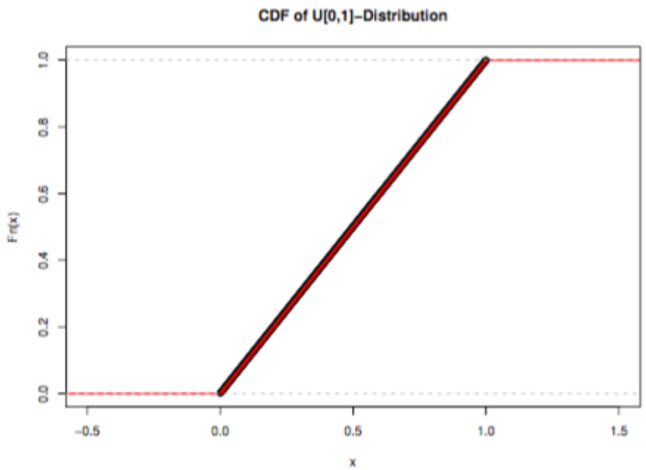
A special case is the **Uniform Distribution on** $[0,1]$ with PDF

$$f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$



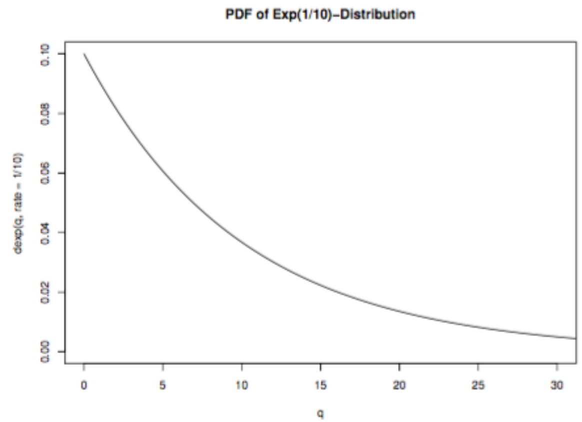
The corresponding CDF is given by

$$F(t) = \begin{cases} 0, & \text{for } t < 0 \\ t, & \text{for } 0 \leq t \leq 1 \\ 1, & \text{for } t > 1. \end{cases}$$



Exponential Distribution with rate lambda

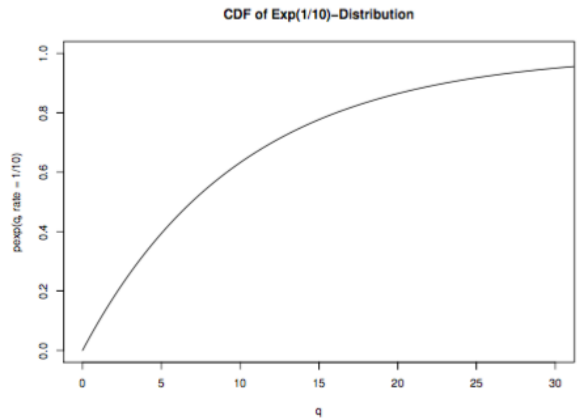
Definition 8 *The distribution of the random variable* $X : \Omega \to [0, \infty)$ *with PDF*



$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

with parameter $\lambda > 0$

is called ***exponential distribution*** *with rate* λ.



The corresponding CDF is given by

$$F(t) = \begin{cases} 0, & \text{for } t < 0 \\ 1 - e^{-\lambda t}, & \text{for } t \geq 0. \end{cases}$$

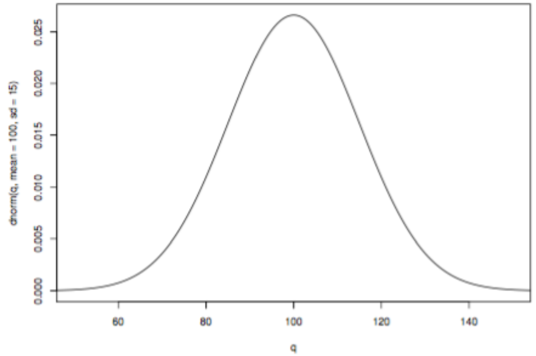
- Example 3: It is 10 minutes before your Statistics lecture starts and you are watching an important match of your favourite hockey team. The game goes overtime. Assuming that the length of overtime until sudden death follows an exponential distribution with rate 1/7 minutes, what is the probability that overtime only lasts for at most 5 minutes (i.e. you'll be in time for your lecture)?

Normal Distribution

Definition 9 *The distribution of the random variable $X : \Omega \to \mathbb{R}$ with PDF*

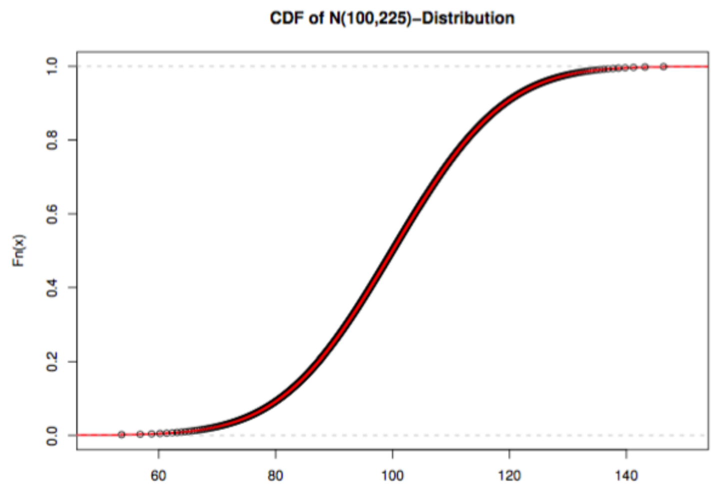
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in \mathbb{R}$$

with $\mu \in \mathbb{R}$ and $\sigma^2 > 0$



*is called **normal distribution** with parameters μ and σ^2.*

There is no closed form expression for the CDF of a normal distributed random variable X



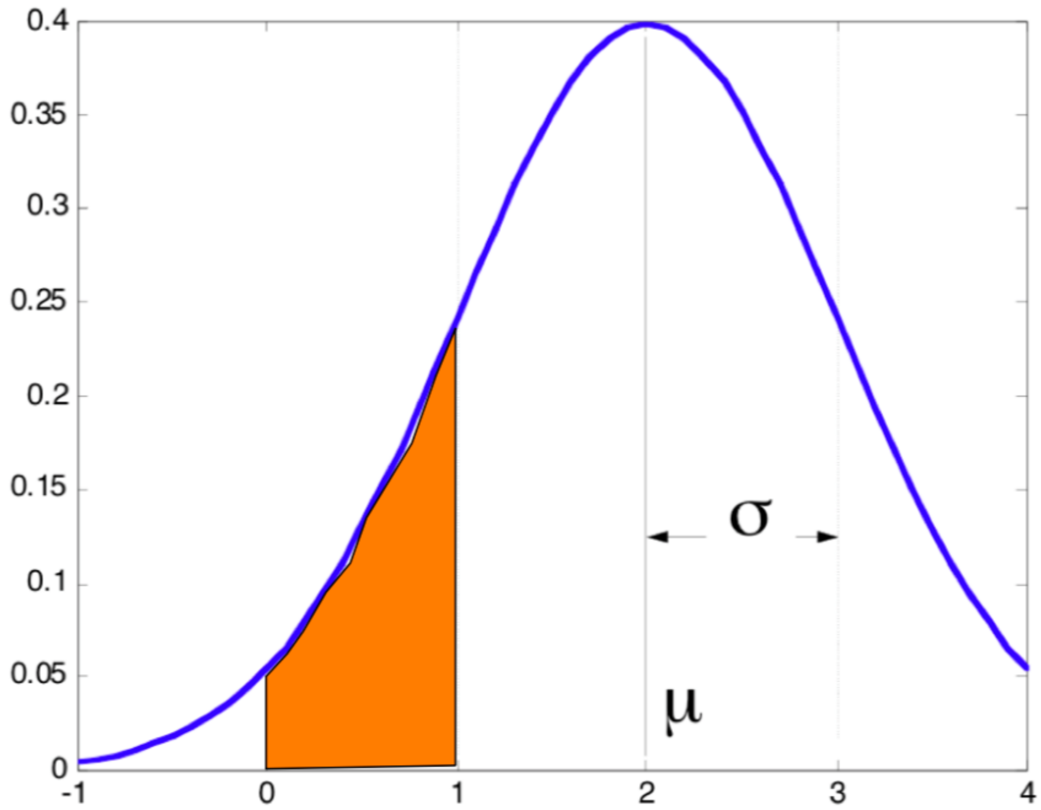


Figure 9: The pdf of the 1-dimensional Gaussian distribution with mean $\mu = 2$ and standard deviation $\sigma = 1$. The orange area gives the probability of the event $[0, 1]$.

	Notation	$F_X(x)$	$f_X(x)$
Uniform	$\text{Unif}(a,b)$	$\begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a<x<b \\ 1 & x>b \end{cases}$	$\frac{I(a<x<b)}{b-a}$
Normal	$\mathcal{N}(\mu,\sigma^2)$	$\Phi(x)=\int_{-\infty}^{x}\phi(t)\,dt$	$\phi(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$
Log-Normal	$\ln\mathcal{N}(\mu,\sigma^2)$	$\frac{1}{2}+\frac{1}{2}\text{erf}\left[\frac{\ln x-\mu}{\sqrt{2\sigma^2}}\right]$	$\frac{1}{x\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(\ln x-\mu)^2}{2\sigma^2}\right\}$
Multivariate Normal	$\text{MVN}(\mu,\Sigma)$		$(2\pi)^{-k/2}\lvert\Sigma\rvert^{-1/2}e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$
Student's t	$\text{Student}(\nu)$	$I_x\left(\frac{\nu}{2},\frac{\nu}{2}\right)$	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-(\nu+1)/2}$
Chi-square	χ^2_k	$\frac{1}{\Gamma(k/2)}\gamma\left(\frac{k}{2},\frac{x}{2}\right)$	$\frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2-1}e^{-x/2}$
F	$\text{F}(d_1,d_2)$	$I_{\frac{d_1x}{d_1x+d_2}}\left(\frac{d_1}{2},\frac{d_1}{2}\right)$	$\frac{\sqrt{\frac{(d_1x)^{d_1}d_2^{d_2}}{(d_1x+d_2)^{d_1+d_2}}}}{x\text{B}\left(\frac{d_1}{2},\frac{d_1}{2}\right)}$
Exponential*	$\text{Exp}(\beta)$	$1-e^{-x/\beta}$	$\frac{1}{\beta}e^{-x/\beta}$
Gamma*	$\text{Gamma}(\alpha,\beta)$	$\frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$
Inverse Gamma	$\text{InvGamma}(\alpha,\beta)$	$\frac{\Gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}e^{-\beta/x}$
Dirichlet	$\text{Dir}(\alpha)$		$\frac{\Gamma\left(\sum_{i=1}^k\alpha_i\right)}{\prod_{i=1}^k\Gamma(\alpha_i)}\prod_{i=1}^k x_i^{\alpha_i-1}$
Beta	$\text{Beta}(\alpha,\beta)$	$I_x(\alpha,\beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$
Weibull	$\text{Weibull}(\lambda,k)$	$1-e^{-(x/\lambda)^k}$	$\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{-(x/\lambda)^k}$
Pareto	$\text{Pareto}(x_m,\alpha)$	$1-\left(\frac{x_m}{x}\right)^\alpha \quad x\ge x_m$	$\alpha\frac{x_m^\alpha}{x^{\alpha+1}} \quad x\ge x_m$

Probability and Statistics Cookbook

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http://statistics.zone/