PSM Fall 2016, Exercise Sheet 3

Return on Thursday October 6 in class

Note. You are encouraged to work in teams of two – but no larger. If you work in a team, submit only a single sheet with both names indicated on it. Nicely type-set solutions are highly appreciated.

Problem 1 (10 points) Let $\Omega = \{\omega_1, \ldots, \omega_6\}$ be a (uncommonly small) universe. Define two RVs $X, Y : \Omega \to \{\text{small, medium, large, x-large}\}$ which are identically distributed but not identical.

Problem 2 (15 points) Let $X_1 : \Omega \to \{1, 2, 3, 4, 5\}, X_2 : \Omega \to \{\text{red, green, blue}\}$ and $X_3 : \Omega \to \{fast, slow\}$ be three RVs with values in $S_1 = \{1, 2, 3, 4, 5, \}$, $S_2 = \{\text{red, green, blue}\}$, and $S_3 = \{fast, slow\}$. Invent and fully specify a joint distribution of these three RVs which makes them jointly independent.

Problem 3 (15 points) Signals that are either 0's or 1's are sent in a noisy communication circuit. The signal received is the signal sent plus a random variable, ϵ, that is normally distributed with mean $\mu = 0$ and standard deviation $\sigma = \frac{1}{3}$. If a 0 is sent, the receiver will record a 0 if the signal received is at most a value v; otherwise a 1 is recorded. Determine v such that the probability that a 1 is recorded when a 0 is actually sent is 0.90.

Problem 4 (15 points) Suppose that $X \sim N(\mu, \sigma)$ and let $Y = e^X$.

1. Find the mean and variance of Y.
2. Find the probability density function of Y. The result is called the *log-normal* probability density function because $\log Y \sim N(\mu, \sigma)$.

Problem 5 (20 points) In an oral exam a student is asked questions until he or she failed to answer three questions. The student fails the exam once she or he answers more questions incorrectly than correctly. Assume that all questions are equally difficult and independent from each other. The probability for a student to give a correct answer to a question is the same for all questions and equals p. Let Y be the total number of questions asked in such an exam.

1. Give a full definition of the random variable Y.
2. Find the probability mass function of Y.
3. Check that the function you have found in subquestion 2 is indeed a probability mass function.
4. For $p = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ compute the probability that the student fails the exam.

Problem 6 (10 points) Let X have a uniform distribution on $[0, 1]$. Find the probability density function for $Y = X^2$ and prove that the result is a probability density function.

Problem 7 (15 points) A professor has two jars of candy on his desk in which originally N candies were filled in each of them. When a student enters her office the student is invited to choose a jar at random and then take a piece of candy from it. At some time one of the jars will be found empty. At the time when one jar is found empty for the first time, how many pieces of candy are in the other jar? Assume that the student chooses with equal probabilities between the two jars.