PSM Spring 2018, Exercise Sheet 2

Problem 1. If $X: \Omega \to \{1, 2\}$, $Y: \Omega \to \{\text{m, f, o}\}$, $Z: \Omega \to \{[0, 10], (10, 100]\}$, what is the size of the sample space of the RV $W = X \otimes Y \otimes Z$ (that is, how many elements does that sample space have)?

Problem 2 (a formalization exercise) Consider a handwriting recognition system which does the job illustrated in Fig. 1 in the lecture notes. The raw input to this system is a high-resolution grayscale photographic image of an entire page of some handwritten document. Assume that this raw input is delivered by a RV $X: \Omega \to$ <the set of all grayscale images of size 2000×1000 pixels>. In a preprocessing stage, from such raw input images small rectangular fields like the one shown in Figure 1 are cropped. In addition to mere cropping, an "area of interest" inside the crop window is identified by the preprocessor. The complement of this area of interest is indicated in Figure 1 by the dark gray background color. Formally, this preprocessor is a transformation operation π which acts on X, giving a new RV $Y = \pi \circ X$. Your task: give a *formal* specification of the sample space of Y (not using plain English but math formalism). Your formalization should take into account the area of interest.

Problem 3 (setting up a model for a hierarchical temporal system) Global economists try to model the global economy system (of course, what else should they do). This is a temporal system of stunning complexity, and modeling it formally as a stochastic process is a difficult task. One difficulty is the heterogeneity of relevant information that has an impact on, or should even be considered part of, the global economical system. These relevant components not only comprise standard financial indicators but also factors like natural catastrophes, wars, elections, inventions... almost everything that happens on this planet.

(a) The easy part of this modeling problem: Describe in English a suitable RSOI. (hint: combine ideas from the Evolutionary Trees I (or II) examples in the LN with what you have learnt about modeling stochastic processes (Section 5 in the LN)). Specifically, what are elementary events ω?

(b) The difficult part of this modeling problem: the RVs X_i and their sample spaces S_i. A comprehensive model of the global economical system would require an extremely large number of very diverse RVs. Since we are dealing with a temporal system, these RVs will mostly (or even all) be time-indexed. However, it is not so straightforward to come up with a good choice for the time index set T. Some observables X will need a fine-grained timescale (for instance sub-second fluctuations of exchange rates in computer trading), others are defined on a daily or monthly grid, yet others are defined for intervals, not points in time (for instance, interest rates fixed by the U.S. Federal Reserve for long times). It wouldn't make much modeling sense to use the finest-grained discrete timescale (or a continuous timescale) to model all of these quantities – e.g. it wouldn't be good modelling craftmanship to define the interest rates fixed by the U.S. Reserve on a second scale.

Your task: think of a *structured* time set T that can accomodate time indices t for the diverse kinds of temporal measurables X_t which we want to include. A simple linear

ordering surely gives not enough structure. Deliverables: (i) a mathematical structure imposed on a time index set T that seems useful for this modeling task; specify this mathematical structure in formalism and explain your underlying ideas in plain English, (ii) some examplary, diverse RVs X_t with their sample spaces S_t, described in plain English (what do they model) and formally (what is the mathematical format of the corresponding S_t).