PSM Spring 2018, homework 4

1. Prove Proposition 9 from the lecture notes.

2. Prove the immediate consequences listed in the LN after Definition 11:

$$\begin{aligned} P(\emptyset) &= 0 \\ P(A^{\mathsf{c}}) &= 1 - P(A) \\ A \subseteq A' &\Rightarrow P(A) \leq P(A') \end{aligned}$$

3. Consider the uniform distribution on the unit interval $S = [0, 1]$. Since this is a part of the real line, this sample space is equipped with the Borel σ-field $\mathfrak{B}([0, 1]) = \sigma(\{(a, b] \mid 0 \leq a \leq b \leq 1\})$. For each interval $(a, b]$ in this generator of $\mathfrak{B}([0, 1])$, we have $P(X \in (a, b]) = b - a$. Use this to show that $P(X = a) = 0$.

4. Show that

$$P(X \in A, Y \in B, Z \in C) = P(X \in A)\, P(Y \in B \mid X \in A)\, P(Z \in C \mid X \in A, Y \in B).$$

5. Show that Equation (17) from the LN follows from Definition 12.