

PSM SPRING 2018, HOMEWORK 5

1. Let (S, \mathcal{F}) be a measurable space and let $A \in \mathcal{F}$ be a nonempty set that is contained in \mathcal{F} . Show that the set $\mathcal{F}_A = \{E \cap A \mid E \in \mathcal{F}\}$ is a σ -field on A .
2. Show that the function P' defined in Equation (12) in the LN is indeed a probability measure.
3. (A study of computer hardware reliability). The CPU of a computer contains a large number, say $L = 10,000$, of *logical gates*. Each such gate is an electronic microcircuit which transforms input bits into output bits. The CPU in its entirety is a clocked device – it updates its state at each *clock cycle*. Today's CPUs have clock cycle rates of about 2 GHz, that is, in one second they go through 2,000,000,000 clock cycles. In each clock cycle, all logical gates are executed (this is a simplification, but not crucial for our argument). Digital computers are very sensitive to bit errors that sometimes — very rarely — occur when a logical gate in the CPU misfires in a clock cycle. Computer hardware engineers spend much care on designing gates in a way that leaves only a very small probability for a CPU gate to make an error (it is impossible to get zero error probability due to the laws of quantum mechanics and thermal fluctuations). Compute the error probability that is admissible for a CPU gate if the requirement is that when the computer runs for an entire day, the probability of having zero gate errors is at least 0.999. Hint: use the approximation $\log(1+x) \approx x$ which is good for small x , and which entails $\log_{10}(1+x) \approx x/\log(10) \approx x/2.3$.