PSM Spring 2018, homework 6

1. (The following probability numbers are my invention)

   - The probability that an adult person is suffering from schizophrenia is 0.01.
   - The probability that an adult person wearing a hearing aid hears voices is 0.1.
   - The probability that a schizophrenic adult person who has a hearing aid hears voices is 0.99.
   - The probability that an adult person has a hearing aid is independent of whether this person is schizophrenic.

   Mr. Thompson, who wears a hearing aid, hears voices. What is the probability that he suffers from schizophrenia?

2. Let $\Omega = \{\omega_1, \omega_2\}$ and $X, Y : \Omega \to \{0, 1\}$. Specify a probability measaure P on Ω and concrete values $X(\omega_i), Y(\omega_i)$ (where $i = 1, 2$) such that X, Y are identically distributed but not for all $\omega \in \Omega : X(\omega) = Y(\omega)$.

3. Verify the claim made in the last bullet point in the list after Definition 12 in the lecture notes.

Solution to 1. With an obvious dirty notation, we compute

$$\begin{aligned} P(S \mid V, H) &= \frac{P(V, H \mid S)\ P(S)}{P(V, H)} \quad \text{;;; Bayes' rule} \\ &= \frac{P(H \mid S)\ P(V \mid H, S)\ P(S)}{P(V \mid H)\ P(H)} \\ &= \frac{P(V \mid H, S)\ P(S)}{P(V \mid H)} \quad \text{;;; independence of H from S} \\ &= \frac{0.99 \cdot 0.01}{0.1} \\ &= 0.099 \end{aligned}$$