PSM Spring 2018, homework 7

1. Show that the function P_X defined in Equation 19 in the LN is indeed a probability measure on $(S, \mathcal{F})$.

2. Let X and Y be two real-valued random variables such that their joint cdf is given by

$$F(x,y) := \begin{cases} 0 & : \text{ if } x < 0 \text{ or } y < 0, \\ x & : \text{ if } 0 \leq x \leq 1 \text{ \& } y > 1, \\ y & : \text{ if } x > 1 \text{ \& } 0 \leq y \leq 1, \\ xy & : \text{ if } 0 \leq x \leq 1 \text{ \& } 0 \leq y \leq 1, \\ 1 & : \text{ if } x > 1 \text{ \& } y > 1. \end{cases}$$

   - What is the pdf of this distribution? (check out the freshly uploaded Version 2.2 of the LN, Section 10.2.2., for 2-dimensional cdf's.)
   - Are X, Y identically distributed? (proof)
   - Are X, Y independent? (proof)

3. *Claim:* "Let X, Y be two real-valued RVs (which admit a representation by pdfs p_X, p_Y) such that $|X(\omega)| > |Y(\omega)|$ for all $\omega \in \Omega$. Then $\mathrm{Var}[X] > \mathrm{Var}[Y]$." Is this claim true or false? (Proof).