PSM Spring 2018, homework 8

1. Show that for two real-valued RVs X, Y with pdf's p_X, p_Y, independence implies uncorrelatedness.

2. Let X, Y be two independent discrete RVs, each taking value in the set $S = \{0, 1, 2\}$, with pmf's $p_X, p_Y : S \to [0, 1]$. What is the pmf of the RV $X + Y$?

3. Let X, Y, Z be three independent RVs, taking value in S_X, S_Y, S_Z. Show that $P(X \in A, Y \in B \mid Z \in C) = P(X \in A \mid Z \in C)\ P(Y \in B \mid Z \in C)$.

4. Let a three-state Markov Chain be given by the transition matrix

$$M = \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0.1 & 0.1 & 0.8 \\ 0.8 & 0.1 & 0.1 \end{bmatrix}$$

   What is the probability that this process ends the third state after three transitions, when it is started in the first state? Compute the solution numerically on your computer.

5. Consider an N-state Markov Chain which has the property that regardless of which starting state is chosen, the probability that a path is found to be in N-th state grows arbitrarily close to certainty, that is

$$\lim_{n \to \infty} P(X_n = s_N \mid X_0 = s_i) = 1$$

   for all $i = 1, \ldots, N$. Give necessary and sufficient conditions on the transition matrix that lead to this behavior (proof not needed – it would require a little more Markov maths than we introduced in the lecture – just want to sharpen your Markov intuitions).