

TMCS Fall 2016, HW 2

Please bring your solutions to class on Thursday September 22. You are invited to work in teams of two but not larger. If you work in a team, submit only a single sheet with both names marked on it.

Problem 1 (20 pts). Consider the following truth table:

X	Y	Z	φ
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Give a Boolean expression for φ in CNF!

Solution. $(X \vee Y \vee \neg Z) \wedge (X \vee \neg Y \vee \neg Z) \wedge (\neg X \vee \neg Y \vee Z)$.

Problem 2 (30 pts). Consider the following facts (well, facts...).

- (a) Everybody who has a good sense of hearing can sing well.
- (b) Nobody is a true musician who cannot inspire her audience.
- (c) Nobody without a good sense of hearing can inspire her audience.
- (d) Nobody except a true musician can write a symphony.

Question: which characteristics must everybody necessarily have who wrote a symphony? First formalize each of the facts (a) – (d) as an implication between two propositions, which gives you four implications, and then use those to find out the answer to the question.

Here is an example of how an English sentence similar to the sentences (a)-(d) above can be cast as an implication:

(x) No stone can fall upwards.

This is turned into an implication by introducing a Boolean variable S to be understood as "is a stone" and a Boolean variable F meaning "can fall upwards", then (x) becomes formalized as $S \rightarrow \neg F$.

For your formalization of (a) – (d), use the Boolean variables H = has a good sense of hearing, S = can sing well, T = is true musician, I = can inspire audience, W = can write symphony.

Solution. Let H = has a good sense of hearing, S = can sing well, T = is true musician, I = can inspire audience, W = can write symphony. Facts (a) – (d) turn into

(a) $H \rightarrow S$

(b) $\neg I \rightarrow \neg T$ (or $T \rightarrow I$)

(c) $I \rightarrow H$

(d) $W \rightarrow T$

The necessary characteristics for having written a symphony are all the properties among H, S, T, I which are entailed by W . Since $W \rightarrow T$, T is entailed, and since W only appears in (d), any other necessary characteristic must be entailed by T . (b) is equivalent to $T \rightarrow I$ (either find this obvious or check by truth table), hence I is another necessary characteristic. By (c) also H is found to be necessary and finally by (a), also S . So, all of H, S, T, I are necessary for a person to have written a symphony.