Your name:

Group A

Comp^2, Spring 2010: Final Exam

Each of the following statements is either true or false. Fill the corresponding answer box with a "T" if you think the statement is true, and with a "F" if you think it is false. If you change your mind and want to flip your first mark, cross it out and write your final judgement besides it in an unambiguous manner. Unfilled or ambiguously marked boxes will be considered as wrong answers.

I designed the questions for this exam with a different strategy than I did in the final of the Formal Languages and Logics lecture. There, all problems were roughly of similar difficulty; here I have a wider spread of difficulty: some problems I would consider very easy, others as more challenging. Questions that I consider easy are marked with a ☺, the more challenging ones are marked with a ✌.

Exam point calculation: There are altogether 30 questions. If you get all of them right, the exam score will be 100. If you get 15 of them right (which amounts to random guesses), the exam score will be 40 points, corresponding to a Jacobs grade of 5.0. If you get N questions right, linear interpolation will be used to obtain the exam points, i.e. the formula exam_points =4 N – 20 *will be used. Plus, there is a bonus question at the end of this sheet.*

Answer box	Statement
T	1. ☺ There are as many recursive functions as there are decidable languages.
F	2. Define a *loop TM* to be a Turing machine whose (single) tape is connected back to itself, forming a loop. Define *loop-decidable* languages to be the languages that can be decided by a loop TM. Claim: the set of loop-decidable languages is the same as the set of the decidable languages.
T	3. ☺ There exists a single-tape TM M with tape alphabet {0,1} which, when started on any input word $w \in \{0,1\}^*$, stops after some time, and when it has stopped, the tape inscription is the binary representation of the length of w.
T	4. There exists a single-tape TM M with tape alphabet {0,1,#} which, when started on any word 1^k, stops; and when it has stopped, the tape inscription is a list of all prime numbers less than k in binary representation, separated by #.
T	5. There exists a single-tape TM M with tape alphabet {0,1,#} which, when started on any word 1^k, stops; and when it has stopped, the tape inscription is a code <*N*> of some TM N with k states, where $N(\varepsilon) = \nearrow$.
F	6. By an interated application of the speedup theorem, every TM M that runs in polynomial time n^k can be accelerated such that it runs in linear time O(n) [albeit possibly at the expense of an explosion of the number of states]

Answer box	Statement
F	7. Let *absdiff*: $\mathbb{N}^2 \to \mathbb{N}$, $absdiff(x, y) = \|x - y\|$. Let μ be the μ-operator defined in Def. 5.3. in the lecture notes. Then, μ *absdiff*(2) is not defined
T	8. Let U be a universal TM. Then for all TMs M, inputs x for M it holds that $U(<M>;<x>) = U(<U>;<<M>;<x>>)$.
F	9. .✌ There exists a number $k \in \mathbb{N}$, such that every decidable language can be decided by some single-tape TM with at most k states.
F	10. ✌ Let $f: \mathbb{N} \to \mathbb{N}$ be a primitive recursive function. Then $f \in$ **P**.
F	11. ✌ Let $f: \mathbb{N} \to \mathbb{N}$ be a recursive function that is not defined on some inputs x. Then there exists an totally defined recursive function $g: \mathbb{N} \to \mathbb{N}$ which has the same values as f on all inputs where f is defined, and a value of 1 on the inputs where f is undefined.
T	12. ☺ Let $w_0 = \varepsilon$, $w_1 = 0$, $w_2 = 1$, $w_3 = 00$, ... be the alphabetic enumeration of $\{0,1\}^*$. For a language $L \subseteq \{0,1\}^*$ let $f_L: \mathbb{N} \to \{0,1\}$, $f_L(i) = 1$ if $w_i \in L$ else $f_L(i) = 0$ be the *characteristic function* of L. Then L is decidable iff f_L is recursive.
F	**13.** It is decidable whether the language $L(M)$ accepted by a TM M is decidable.
T	14. ☺ The language $L_{\text{Boole}} = \{<\varphi> \mid <\varphi> \text{ is the code of an unsatisfiable Boolean formula}\}$ is decidable.
F	15. If **P = NP** would hold, then it would follow that first-order logic is decidable.
F	16. In combinatorial algebra, $K(K(K))$ reduces to K.
T	17. In combinatorial algebra, there exist infinitely many different terms made exclusively of S's and K's (plus, of course, possibly brackets) which are all equivalent to I.
T	18. ☺ The expression $(\lambda z.(xx))$ is a well-formed λ-term.
F	19. **tail (cons head tail) →* nil**
T	20. $\underline{2}\, f\, (Yf) \equiv Yf$
F	21. A *decision problem* in the sense of complexity theory can have *instances* for which a yes/no answer is not defined.
T	22. ✌ Let $R \subseteq \Sigma^* \times \Sigma^*$ be a polynomially decidable relation. Furthermore, assume that R is *logarithmically balanced*, that is, $(x, y) \in R$ implies $\|y\| \leq \log\|x\|$. Let $L = \{w \mid (w, y) \in R \text{ for some } y\}$. Then $L \in$ **P**.
T	23. TSP(D) $\in$ **PSPACE**
T	24. **NPSPACE** contains only decidable languages.
F	25. The configuration graph $G(M, x)$ of a *deterministic* TM M on input x is free of cycles.
T	26. ☺ **NP** $\subseteq$ **PSPACE**
T	27. SAT is **P**-hard with respect to ptime-reductions.
F	28. If L is **NP**-hard and L can be ptime-reduced to L' and $L' \in$ **P**, then **P ≠ NP.**
T	29. ☺ TSP(D) is **NP**-complete.
F	30. ✌ There exist infinitely many regular languages which are **P-**

	complete.

Optional bonus question (estimated processing time: 15 minutes) *For this bonus question, a maximum of 10 points is added to the point score obtained from the multiple-choice part of this exam. If in this way you reach an exam score above 100, it will be counted toward the course grade.*

If you use separate sheets for the bonus question, make sure your name is indicated on each sheet.

Prove the following statement: The language

L = {<*M*>;<*N*> | <*M*> is a code of a TM *M* with tape alphabet {0,1},

<*N*> is a code of a TM *M* with tape alphabet {0,1}, and

for all $x \in \{0,1\}^*$ it holds that $M(x) = N(x)$}

is undecidable.