

Reservoir Riddles: New Directions in ESN Research

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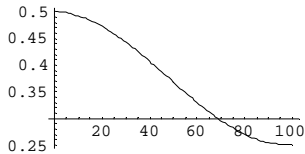
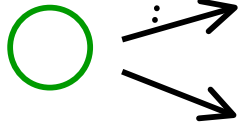
Overview

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2. Current R&D at IUB/Fraunhofer
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 2. Optimization of global control parameters
 3. Continuous-time ESNs
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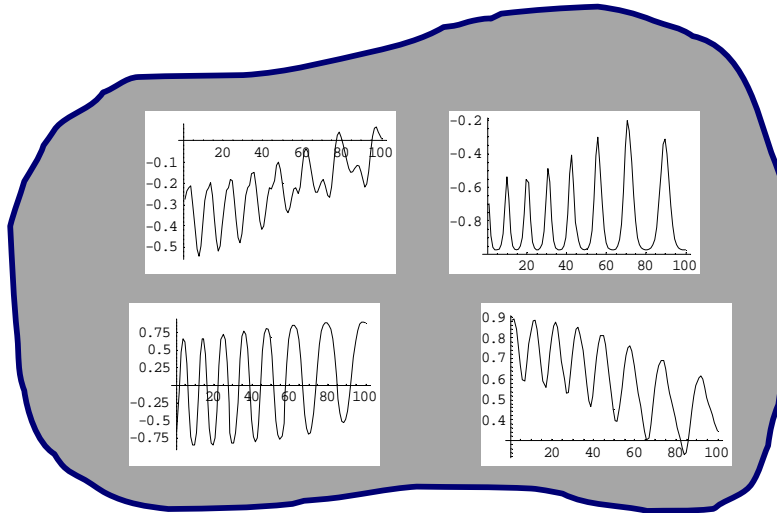
1. The ESN/LSM principle

Input weights:

- Fixed
- Random



Input signal

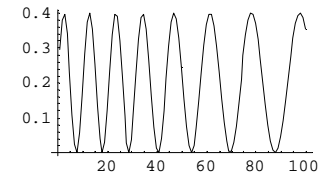
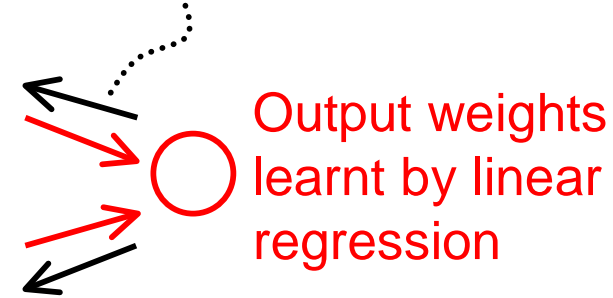


"Dynamical Reservoir":

- Large
- Fixed
- Random
- Recurrent

Output feedback weights (optional):

- Fixed
- Random



Output (teacher) signal

2 Current R&D

2.1 Learning to train ESNs

Problem: novices take weeks to months to achieve good ESN training results, or quit prematurely

Reason: it's apparently not so easy to get a "gut feeling" for excited nonlinear dynamics

- judge degree of nonlinearity
- judge short-term memory requirements
- negotiate on the bias-variance scale, master noise regularization

Non-solution: refer novice to techreport¹⁾ with "tricks of the trade"

Solution: develop Matlab tutorial with didactic exercise cases

1) <http://www.faculty.iu-bremen.de/hjaeger/pubs/ESNTutorialRev.pdf>

2.2 Optimizing global control parameters

Problem: network size, spectral radius, input scaling, input shift, noise level are important global learning controls

How ESN experts "solve" it: tuning by hand

Novices: can't do it that way!

Better solution: automated search for locally optimal global parameters by empirical gradient descent (student project)

Desirable solution: thorough theoretical understanding (open research issue)

2.3 Continuous-time ESNs

Problem: discrete-time OOMs don't work well for slow dynamics

Approach: use (Euler approximation of) leaky integrator neuron ESN, where $\Delta = \Delta t/c$, a leaking rate, c time constant:

$$\mathbf{x}(n+1) = (1 - a\Delta)\mathbf{x}(n) + \Delta f(\mathbf{W}_{in}\mathbf{u}(n+1); \mathbf{W}\mathbf{x}(n))$$

Difficulty 1: guarantee echo state property

Difficulty 2: prevent sigmoid f from saturation

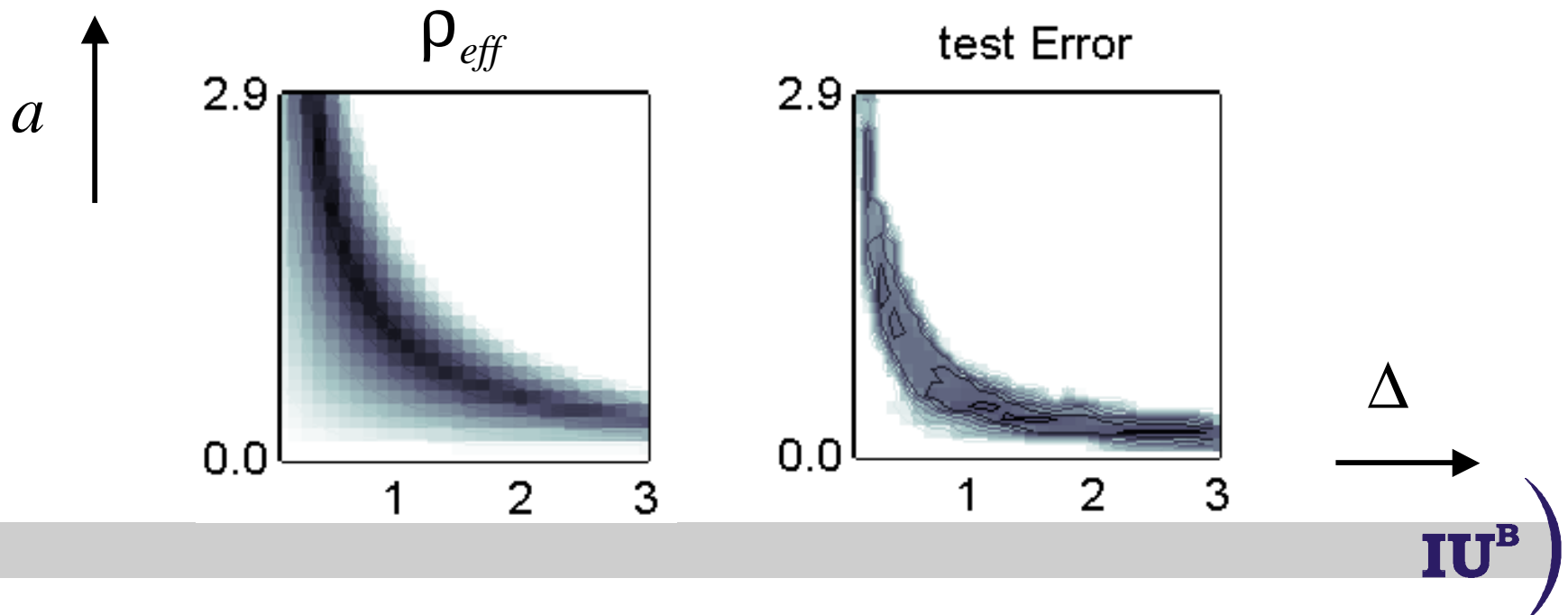
Solution: observe algebraic constraints detailed out in techreport¹⁾

1) <http://www.faculty.iu-bremen.de/hjaeger/pubs/ESNTutorialRev.pdf>

Difficulty 3: we must tune two more global control parameters (leaking rate a , ratio $\Delta = \Delta t/c$)

Facilitation: effects of a and Δ can in practice be combined into single "effective spectral radius"

$$\rho_{eff} = \text{specrad}(\Delta \mathbf{W} + (1 - a\Delta)\mathbf{I})$$



2.4 Coping with time-warped data

Problem: in handwriting and speech recognition, target patterns occur in time-warped variants

Approach¹⁾: use (Euler-discretized) continuous-time ESNs, adapt old idea²⁾: adjust Δ dynamically according to metric input distance:

$$\Delta_n = K \|\mathbf{x}(n) - \mathbf{x}(n-1)\|$$

Difficulty 1: how to choose the scaling constant K ?

Heuristic solution: first optimize for fixed Δ , then set K such that $E[\Delta_n] = \Delta$.

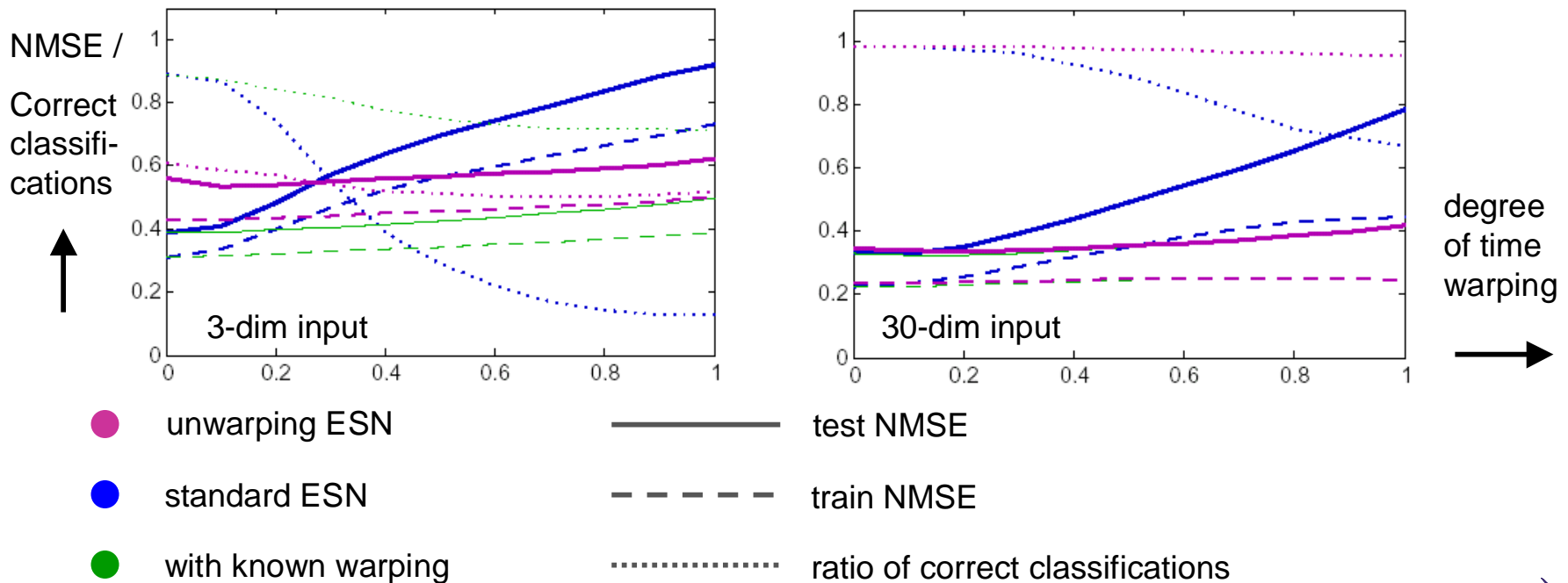
1) Joint project with Planet AG, Raben-Steinfeld, Germany

2) G-Z Sun, H-H Chen, Y-C Lee. Time warping invariant neural networks. NIPS 5 (1993)

Difficulty 2: stiff input dynamics \rightarrow very large $\Delta_n \rightarrow$ unstable (Euler discretized) network dynamics

Heuristic solutions: soft-bound Δ_n by wrapping with sigmoid, or interpolate high-slope input segments

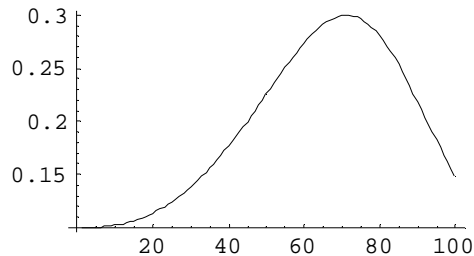
Result¹⁾: (on synthetic dynamic pattern detection task)



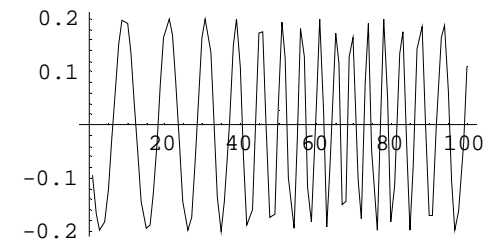
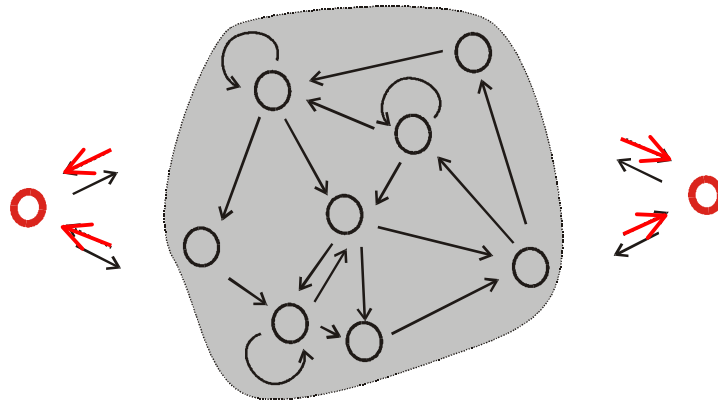
1) Lukosevicius / Popovici / Jaeger / Siewert, Time Warping Invariant Echo State Networks (subm.)

2.5 Bidirectional dynamics: frequency generating/measuring device

Teaching signals: sine wave and its frequency (N = 1000, here section with N = 100)

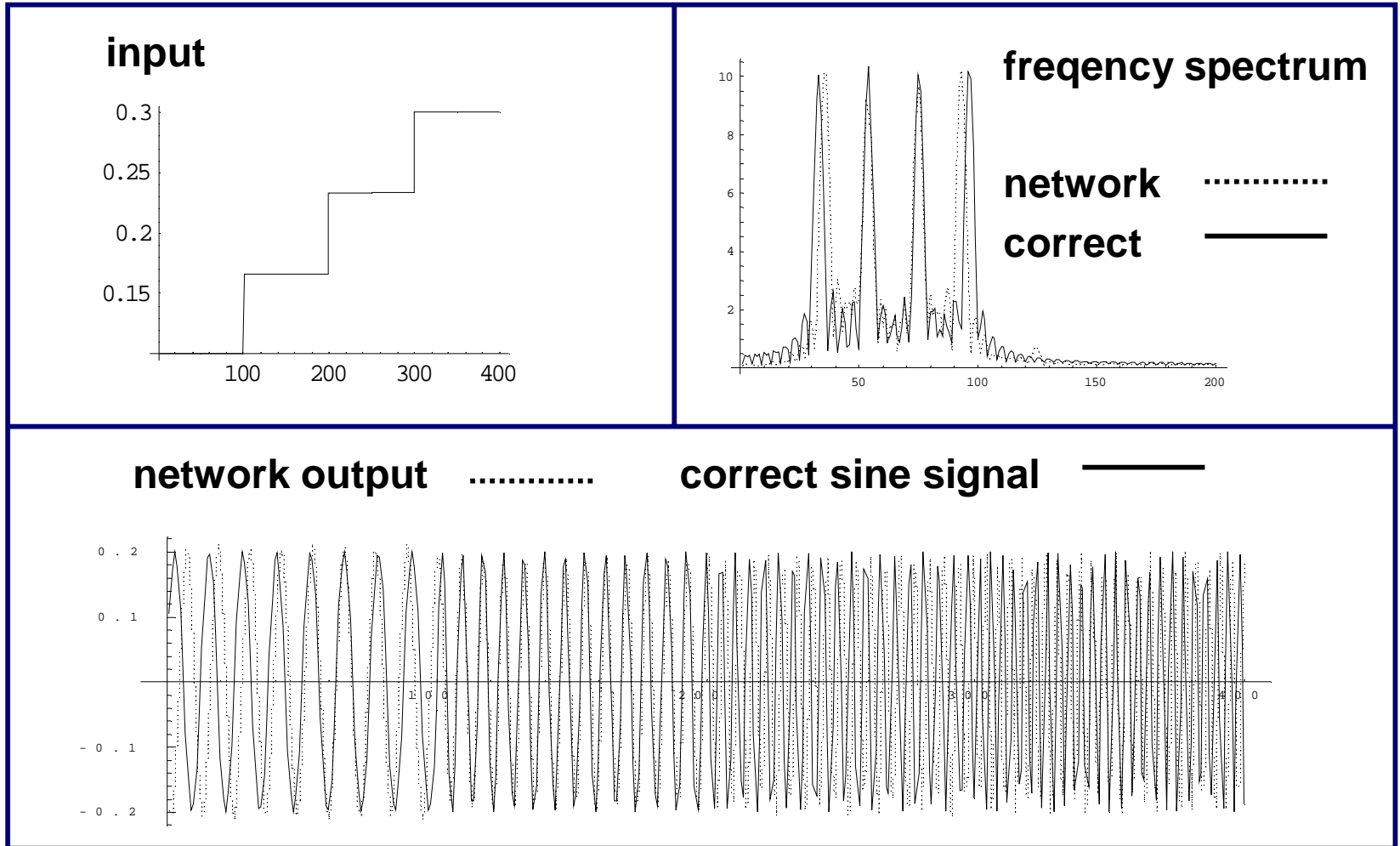


signal 1: frequency



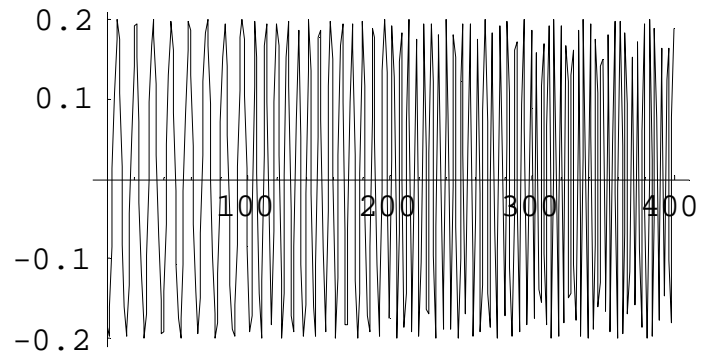
signal 2: sines

Trained ESN used as frequency generator:



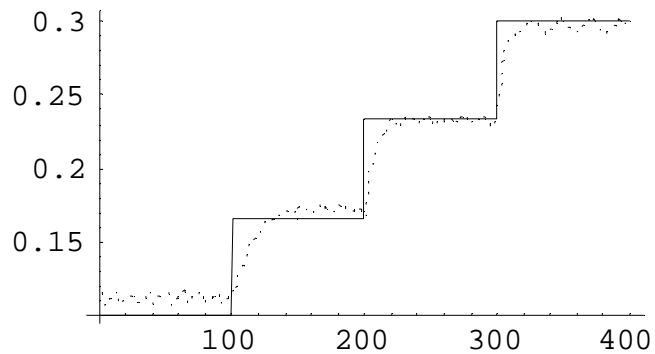
Trained ESN used for **frequency measurement**:

input



network output

correct frequency ———

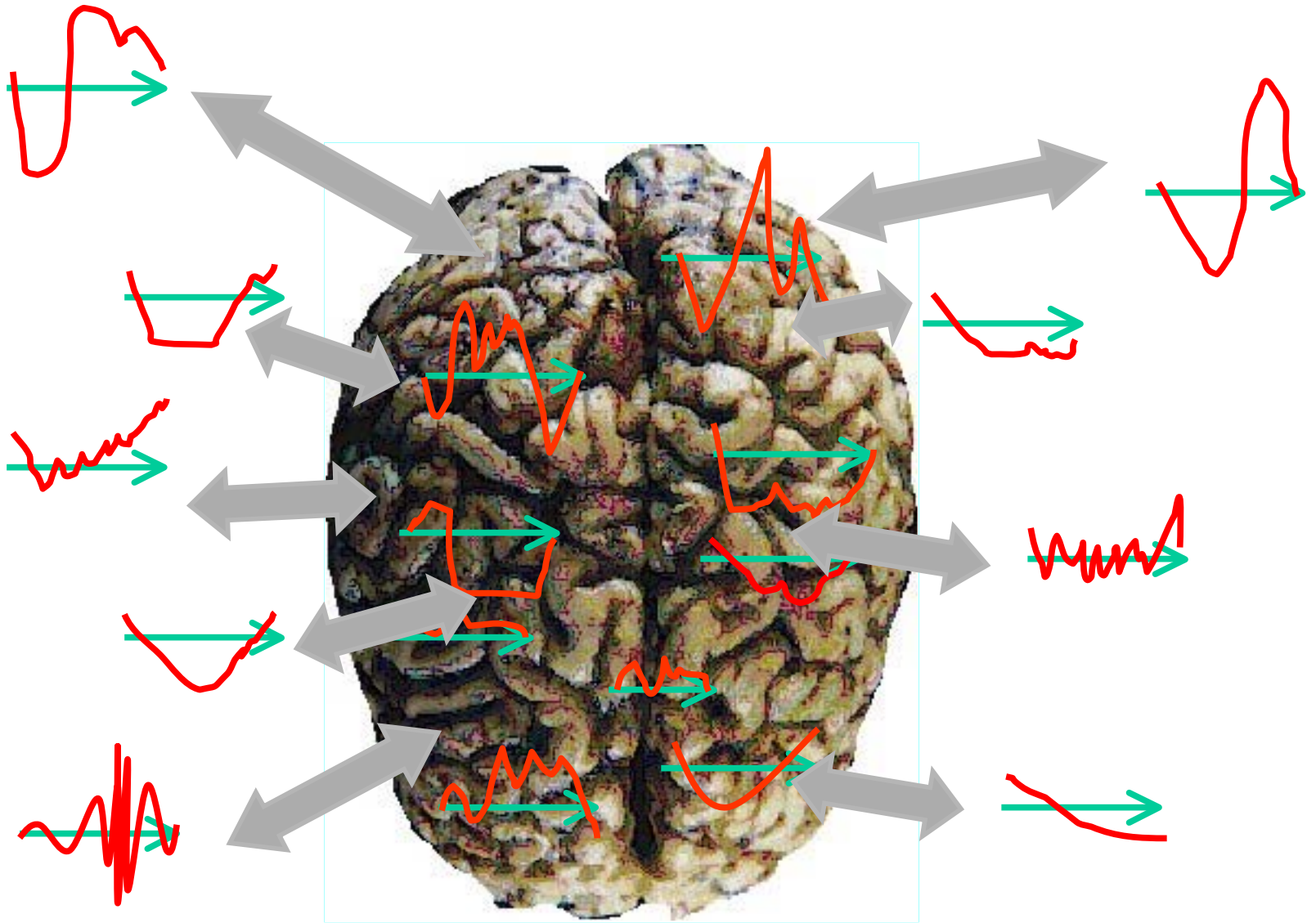


Also tried: bidirectional controller / predictor device

In our brains: multi-directional, cross-modal association of dynamic patterns, e.g.

- visual display of dance ↔ music score
- limb motion (proprioceptive) ↔ limb motion (motor command) ↔ music score
- listening to speech ↔ watching lip motion
- motor command generation ↔ expectation of proprioceptive feedback
- coordinating actuators
- etc.

3 The Riddle



3.1 The problem(s)

Problem 1: On some problems (ask Danil), ESN models inferior to BPTT or EKF models.

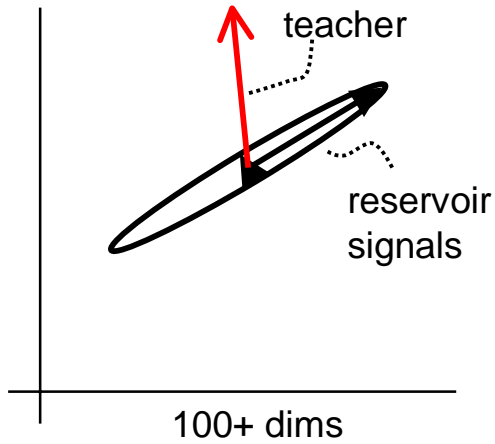
Reservoir doesn't offer the requisite signal components.

Problem 2: On some (deterministic) problems, trained output weights grow very large.

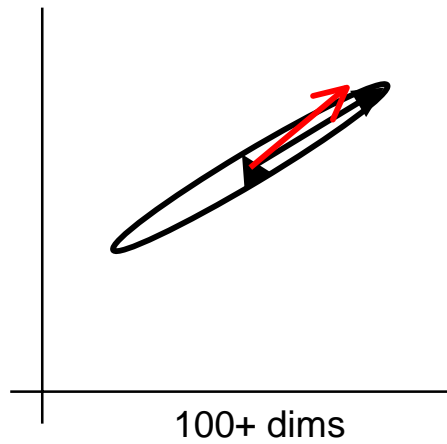
Relevant reservoir signal components are in "thin" directions of the linear reservoir signal space.

Problem 3: The reservoir signal correlation matrix $(E[x_i x_j])$ typically has an extreme eigenvalue spread (order of $1E14$ and higher) \rightarrow impossible to use cheap online learning by LMS, impossible to use analog VLSI or integer arithmetics for reservoir.

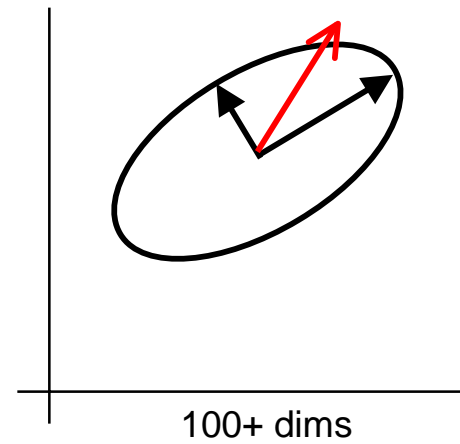
Reservoirs tend to have very similar unit signals due to mutual entrainment of unit's dynamics.



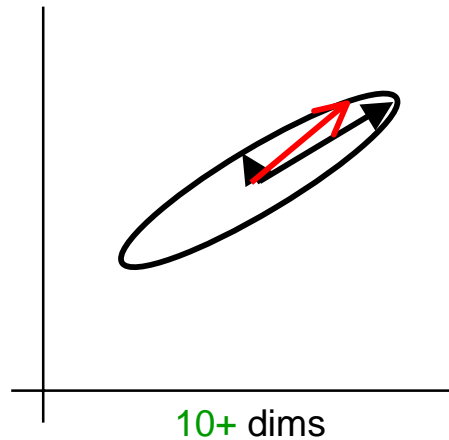
What we get



What we want...



What we want even more...



What BPTT or EKF gives us

Wish, the first: ESN reservoir signal space with **flat spectrum** (many orthogonal reservoir signal components of similar power),

and / or

wish, the second: ESN reservoir signal space which neatly **embraces teacher signal** (small teacher component orthogonal to reservoir signal span).

3.2 Non-solutions for flat spectrum

- Unsupervised "preshaping" of reservoir by anti-Hebbian learning
Spectrum stays same or even spreads further!
- Unsupervised "preshaping" of reservoir by Hebbian learning
Spectrum spreads further!
- Use non-random reservoir connection graphs (scale-free, small-world (= clustered), hierarchical, grid topology...)
No significant effects on spectrum found!¹⁾
- Train reservoir neurons (with ESN rule) individually to orthogonal signals
Doesn't work -- training one neuron disrupts training results of previously trained ones

1) Benjamin Liebald: Exploration of effects of different network topologies on the ESN signal crosscorrelation matrix spectrum. Guided Research Thesis, IUB 2004

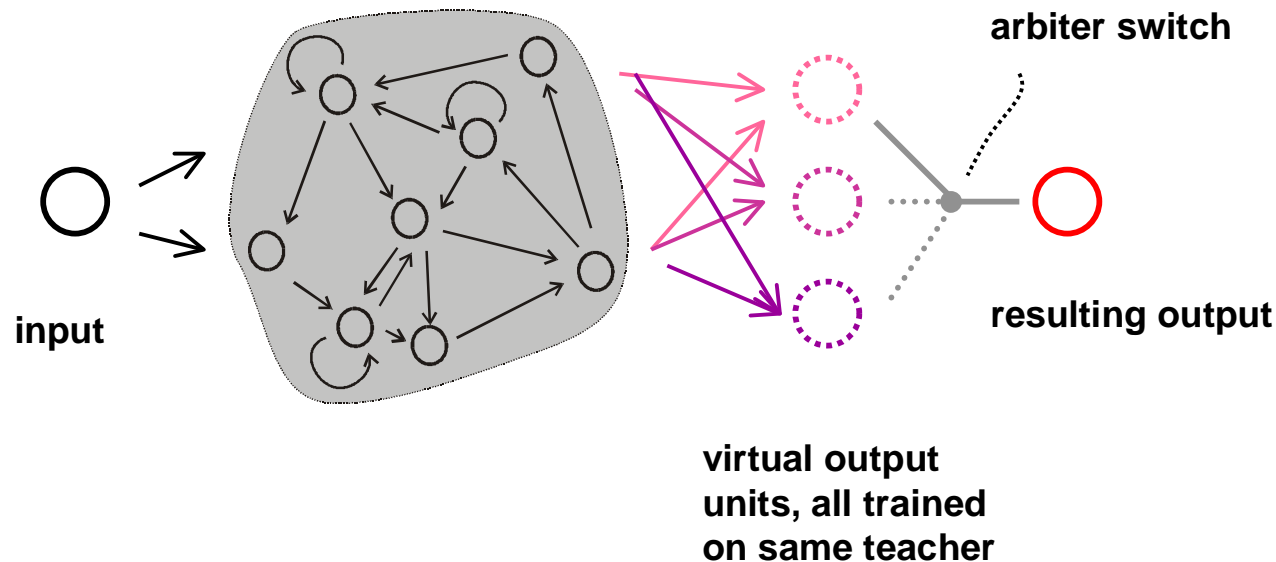
Non-solutions for embracing teacher signal in reservoir dynamics

- Evolutionary algorithms, "traditional" learning algorithms like BPTT or EKF

Defies raison d'être for ESNs!

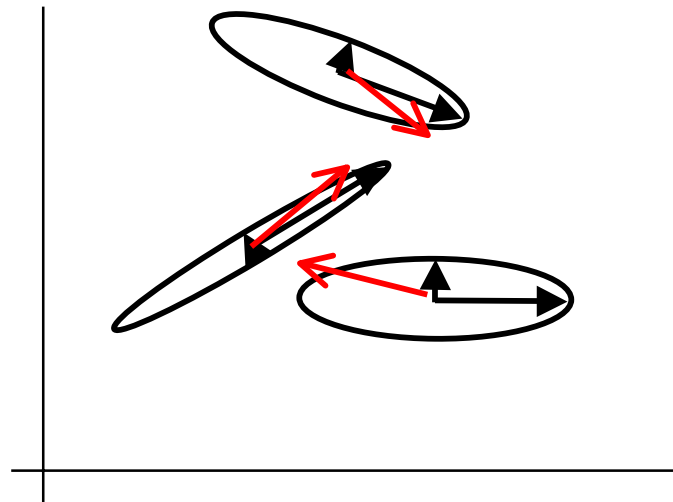
3.3 A partial solution

Idea: train several sets of output weights, switch between them by arbiter



Switching criteria: reservoir state partitioning, input partitioning, history-dependent criteria... Or train virtual output units competitively (did not work well so far)

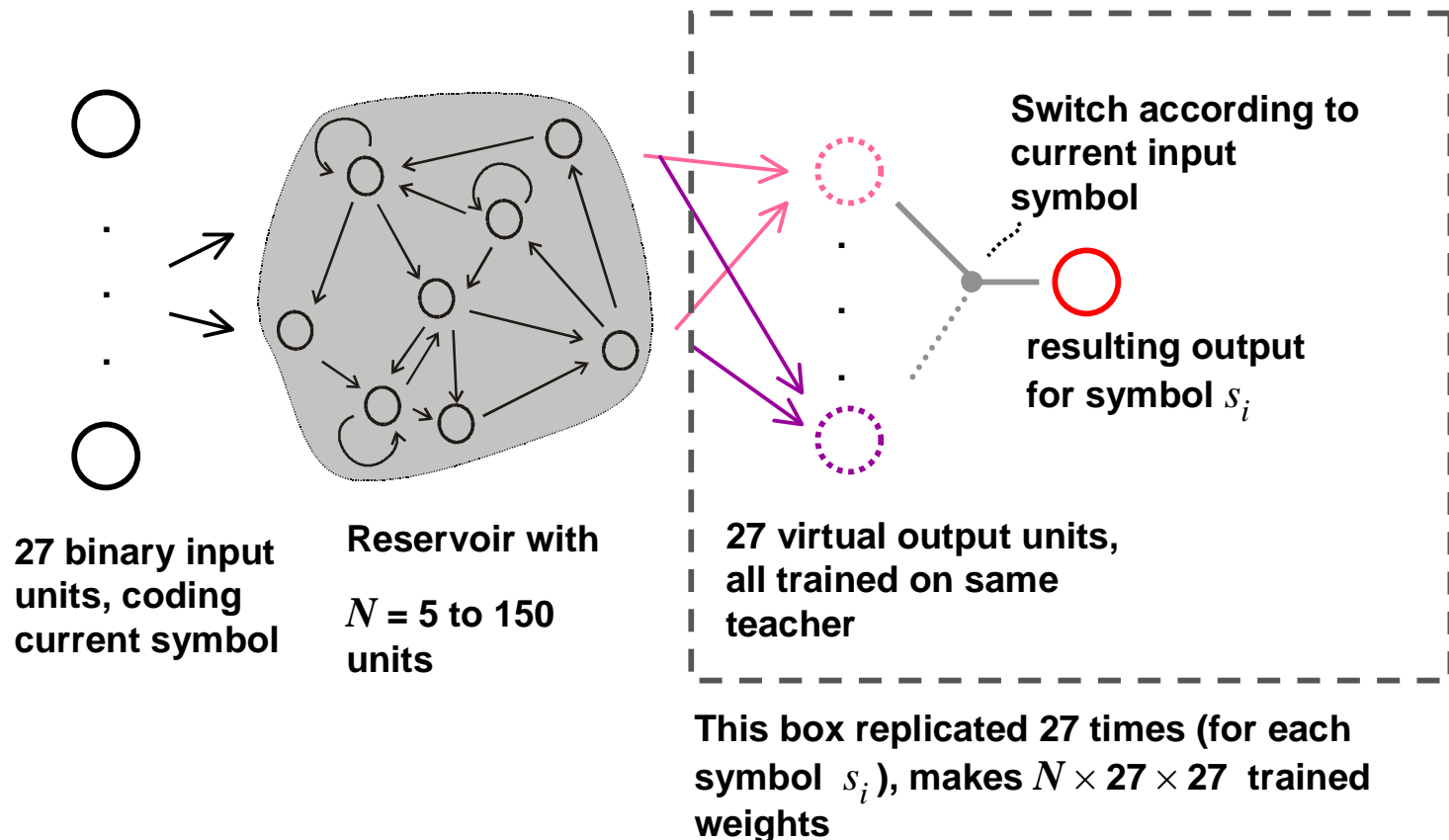
This decomposes reservoir and teacher signals in time -- has aspects of both flattening spectrum and embracing teacher signal.



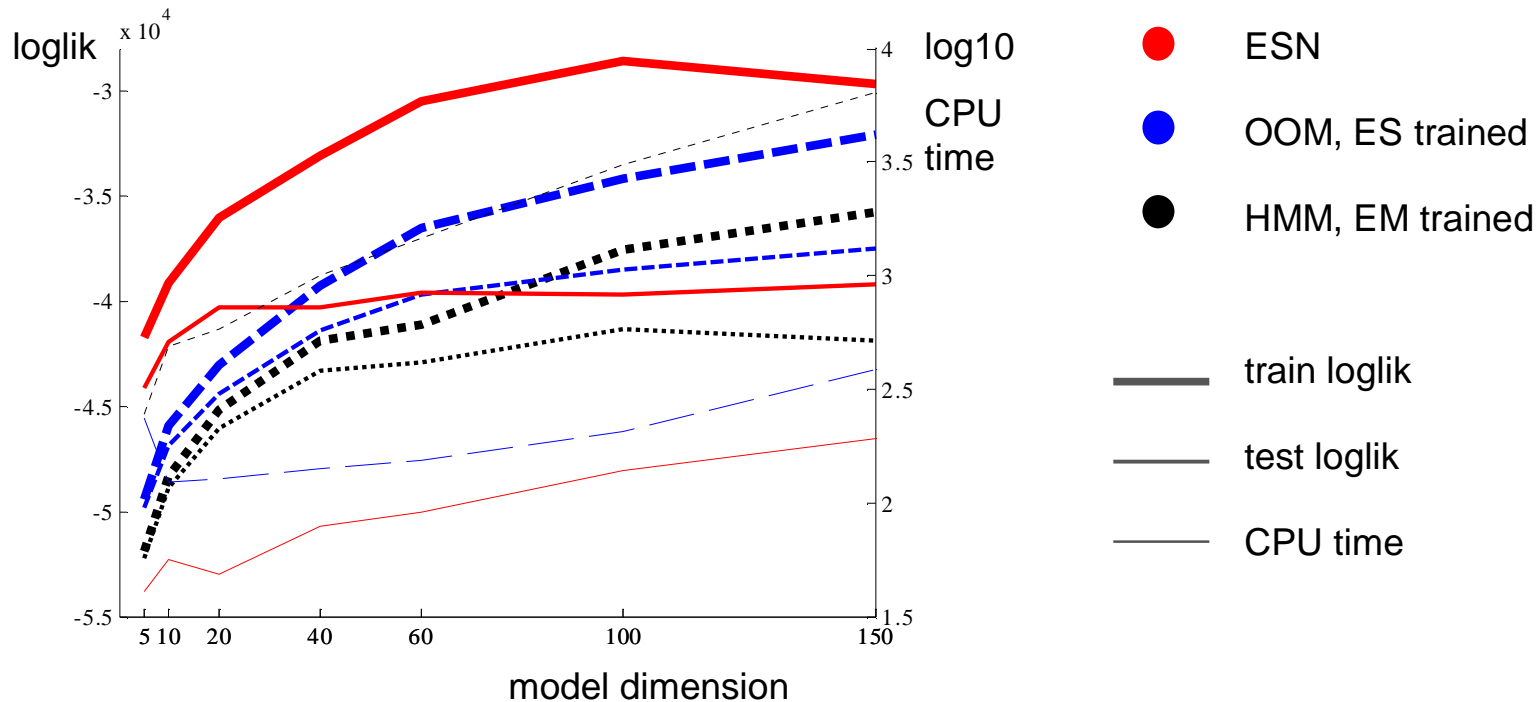
Case study: learning probability distribution of a belletristic text

Data: $2 \times 21,000$ step 27-symbol sequence from Mark Twain short story

Task: Prediction of next symbol (yields distribution for next symbols)



Results



- ESN test performance better than HMM
- Best ESN test loglik reached with 20 dim reservoir, $20 \times 27 \times 27 = 14,580$ trained weights (best HMM dim 100 = 12,700 parameters)
- 2 orders of magnitude shorter learning time than HMM

3.4 Some Speculation

Assumption: biological (vertebrate?) brains use ESN principle at some places

Riddle: how is flat spectrum / teacher embracement accomplished?

Speculation:

- component signals come from specialized and spatially separated regions
- global brain network structure (small-world, hierarchic, clustered, spatially organized) correlates with hierarchy of \pm orthogonal feature detectors
- evolutionary pressure toward formation of \pm orthogonal feature detectors
- no reservoir -- output unit distinction: brain = reservoir = trainable units

Suggestion for artificial networks:

- complex architectures needed: complex network topology, local learning
- no simple trick with \pm homogeneous reservoir

Thank you.

(Interested in test-using an ESN Matlab toolbox?

Then tell me your contact coordinates right here.)