Biomedical Signal Processing

Data Engineering Program, spring 2018

Exercise Sheet 1

February 27, 2018

Submission until March 13th, 24.00 to f.hadaeghi@jacobs-university.de

Exercise 1) Simple filters

1-1. A discrete- time signal $x[n]$ is shown in Figure P1-1. Sketch and label the following signals.

1-1.a $x[n-2]$

1-1.b $x[4-n]$

1-1.c $x[2n]$

1-1.d $x[n-1]\delta[n-3]$

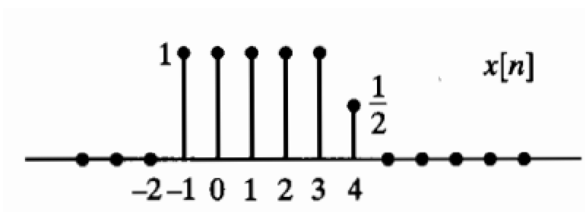
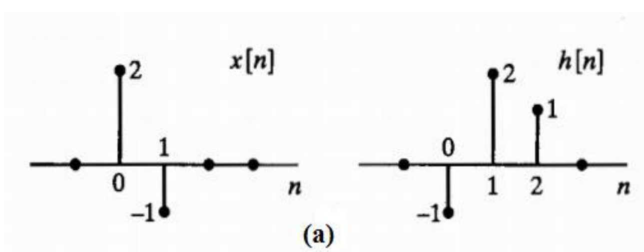
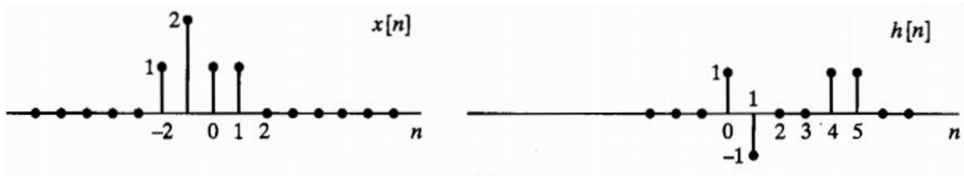


Figure P1-1.

Exercise 2) Convolution sum

2-1. For each of the pairs of sequences in Figure P2-1, use discrete convolution to find the response to the input $x[n]$ of the linear time-invariant system with impulse response $h[n]$.





(b)

Figure P2-1.

Exercise 3) Frequency Response of LTI systems

3-1. Indicate which of the following discrete-time signals are eigenfunctions of stable, linear time-invariant discrete-time systems:

3-1-a. $e^{2\pi nj/3}$	3-1-b. 3^n	3-1-c. $2^n u[-n-1]$
3-1-d. $\cos(\omega_0 n)$	3-1-e. $(1/4)^n$	3-1-f. $(1/4)^n u[n] + 4^n u[-n-1]$

3-2-a. Find the frequency response of the LTI system whose input and output satisfy the difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

3-2-b. write a difference equation that characterizes a system whose frequency response is:

$$H(e^{j\omega}) = \frac{1-\frac{1}{2}e^{-j\omega}+e^{-3j\omega}}{1+\frac{1}{2}e^{-j\omega}+\frac{3}{4}e^{-2j\omega}}$$

3-3. a. Determine the Fourier transform of the sequence

$$r[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & otherwise \end{cases}.$$

3-3. b. Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left[1-\cos(\frac{2\pi n}{M})\right] & 0 \le n \le M \\ 0 & otherwise \end{cases},$$

express the $W(e^{j\omega})$, the Fourier transform of $w[n]$, in terms of $R(e^{j\omega})$, the Fourier transform of $r[n]$.

Exercise 4) The output of the LTI system

4-1. Determine the output of a linear time-invariant system if the impulse response $h[n]$ and the input $x[n]$ are as follows:

a) $x[n] = u[n]$ and $h[n] = a^n u[-n-1]$, with $a > 1$

b) $x[n] = u[n-4]$ and $h[n] = 2^n u[-n-1]$

4-2. Consider an LTI system with frequency response

$$H(e^{j\omega}) = e^{-j(\omega - \frac{\pi}{4})} \frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}}, \qquad -\pi < \omega < \pi.$$

Determine the output $y[n]$ for all n if the input for all n is $x[n] = \cos(\frac{\pi n}{2})$.

4-3. Consider the system in Figure P4-2.

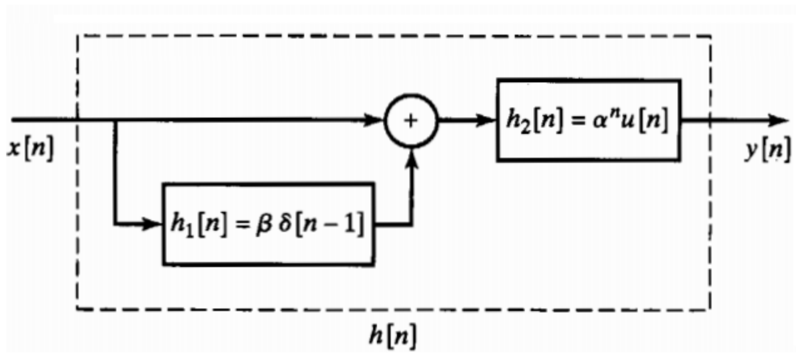


Figure P4-2.

(a) Find the impulse response $h[n]$ of the overall system.

(b) Find the frequency response of the overall system.

(c) Specify a difference equation that relates the output $y[n]$ to the input $x[n]$.

(d) Is this system causal? Under what condition would the system be stable?