

Exercises for Computability and Complexity, Spring 2018, Sheet 1

Please return your solutions in the Thursday lecture on Feb 15, in class. You may work in teams of 2 (hand in one solution sheet for the team)

Exercise 1 Give a transition table for a TM that computes the function $f(n) = 2n$. The TM should have the tape alphabet $\{0, 1, \triangleright, \sqcup\}$ and numbers are coded as binary strings by writing them to base 2.

Exercise 2 If one would admit TMs with countably infinitely many states, would this extend the set of TM-computable functions on the integers? In other words, is there a function $f: \mathbb{N} \rightarrow \mathbb{N}$ which can be computed by some TM with countably infinitely many states, but not by any ordinary TM? Sketch a proof for your answer.

Exercise 3 (deferred to exercise sheet 2).

Exercise 4 Show that $L = \{w \in \{1\}^* \mid |w| \text{ is a power of } 2\} \in \mathbf{TIME}(O(n \log n))$, by describing in words (and maybe sketches of interesting configurations) a TM (with possibly several tapes) that does this job.