Exercises for Computability and Complexity, Spring 2018, Sheet 4

Please return on Thursday, March 8, in class. As usual you are invited but not requested to work in teams of size at most 2.

Exercise 1 Consider the set ***T*** of all single-tape TMs with tape alphabet $\Sigma = \{0, 1, \sqcup, \rhd\}$. Design a coding scheme by which every TM *M* in ***T*** becomes coded by a codeword <*M*> $\in \{0, 1, \#\}^*$. Describe your coding scheme in formal notation and use it to encode the the ultra-simple TM *M* with tape alphabet $\Sigma = \{0, 1, \sqcup, \rhd\}$ and states $\{s, yes, no\}$ that has the following transition table:

$p \in K$	$\sigma \in \Sigma$	$\delta(q, \sigma)$
s	0	$(yes, 0, -)$
s	1	$(s, 1, \rightarrow)$
s	$\sqcup$	$(no, \sqcup, -)$
s	$\rhd$	$(s, \rhd, \rightarrow)$

Exercise 2 (rather easy) Prove that H_2 = {<*M*>;*x* | *Code*(<*M*>) and *Standard*(*x*) and there exists some *y* with *Standard*(*y*) such that *M*(*x*) = *y*} from Proposition 6.3 is undecidable.

Exercise 3 (medium) Show that the language

L = {<*M*> $\in \{0, 1, \#\}^*$ | *M* halts on no input}

is not recursively enumerable. *Hint: in addition to a reduction argument, you might wish to also work in Proposition 3.1 from the lecture notes.*

Challenge problem (optional, not easy) Prove the following claim: If *L* is recursively enumerable but not recursive, then there exists another language *L'* which is likewise r.e. but not recursive, such that $L \cup L'$ is recursive.