

Exercises for Comp & Comp, Spring 2019, Sheet 10

Please return Tuesday May 7 in class.

Problem 1 (rather easy). Prove or disprove the following claim:

Let $R \subseteq \Sigma^* \times \Sigma^*$ be a polynomially decidable relation. Furthermore, assume that R is *constant balanced*, that is, there exists a constant C such that $(x, y) \in R$ implies $|y| \leq C$. Let $L = \{w \mid (w, y) \in R \text{ for some } y\}$. Then $L \in \mathbf{P}$.

Problem 2 (easy). Claim: $\text{SPACE}(1) = \text{TIME}(1)$. Prove or disprove.

Problem 3 (needs an idea, but is rather straightforward). The problem SET PACKING has instances consisting of a finite collection C of finite sets and of a positive integer $K \leq |C|$. The question to be decided is whether C contains at least K disjoint sets. – The problem CLIQUE has instances consisting of an undirected graph $G = (V, E)$ and a positive integer $K \leq |V|$. The question to be decided is whether G contains a *clique* of size at least K , that is, a subset $V' \subseteq V$ such that every two vertices in V' are joined by an edge in E . Reduce SET PACKING to CLIQUE. Don't forget to demonstrate that your reduction can be done in polynomial time!