Exercises for FLL, Fall 2018, sheet 10 - solutions

Return Thursday Nov 22, in class.

Exercise 1. At the end of this exercise sheet I append an old exercise from 2004 together with its solution. That old exercise outlines in its problem statement how DFAs can be seen as *S*-structures, and in the model solution shows how they can be framed in FOL axioms. Use the old exercise and its solution as a source of inspiration and do the same for context-free grammars in Chomsky normal form! that is, give a similar outline of such grammars as *S*-structures, and provide FOL axioms. Do not try however to code the requirement that every symbol in a CNF grammar must be useful – that would be quite involved. *Note*: it is *much* more difficult to axiomatise the context-free grammars of arbitrary form – a challenge for the very ambitious ones!

Solution. A grammar can be seen as a structure G = $(A, V^A, T^A, s^A, P_1^A, P_2^A)$, where the carrier *A* consists of variables and symbols, *V* is a unary predicate symbol (to single out the variables), *T* is a unary predicate symbol (to single out the terminals), *s* is a constant (for the start variable), P_1 is a binary relation symbol (to hold between variables and terminals, to capture the $A \to a$ type productions), and P_2 is is a ternary relation symbol (to hold between variables, to capture the $A \to BC$ type productions).

Axioms:

$\forall x\ ((Vx \lor Tx) \land \neg\ (Vx \land Tx))$	every thing must be either a variable or terminal
$(\exists x\ Vx \land \exists x\ Tx)$	variable and terminal sets are not empty
Vs	the start variable is a variable
$\forall x \forall y\ (P_1 xy \to (Vx \land Ty))$	the P_1 relation is between variables and terminals
$\forall x \forall y \forall z\ (P_2 xyz \to ((Vx \land Vy) \land Vz))$	the P_2 relation is between variables

The case of arbitrary form grammars is so difficult because bodies of rules can have any finite length. One approach that I have in mind (didn't work it out though) is to include in the carrier *A* a subset *B* made of rule bodies and their suffixes, introduce a partial ordering on this set (which captures the length of bodies / suffixes), and make *B* understandable as made of *strings* over terminals and variables by introducing for every terminal and variable symbol X a ternary relation F_X, such that exactly one such F_X holds between a body (or suffix) and its direct successor in the ordering (ie. its suffix)...

************* attached: the similar problem from a 2004 exercise sheet **********

Old exercise from 2004: A DFA can be seen as a structure $\mathcal{D} = (A, S^A, Q^A, F^A, \delta^A, q_0{}^A)$, where the carrier *A* consists of the states and symbols, *S* is a unary predicate (intention: *S* denotes the symbols), *Q* is a unary predicate (denoting the states), *F* is a unary predicate (denoting the accepting states), δ is a binary function (denoting the transition function), and q_0 is a constant symbol (denoting the start state). Give a collection Φ of FOL propositions such that every finite *S*-structure $\mathcal{D}$ is a model of Φ iff $\mathcal{D}$ corresponds to a DFA. In other works, axiomatize the DFAs in FOL. Explain each of your propositons in words.

Solution (to the 2004 homework problem, serves as a hint). Here is one possibility:

$\forall x\, ((Sx \vee Qx) \wedge \neg\, (Sx \wedge Qx))$	every thing must be either a state or a symbol
$(\exists x\, Sx \wedge \exists x\, Qx)$	state and symbol sets are not empty
Qq_0	the start state is actually a state
$\forall x\, (Fx \rightarrow Qx)$	the accepting states are actually states
$\forall x \forall y \forall z\, (((Qx \wedge Sy) \wedge \delta xy = z) \rightarrow Qz)$	δ maps state-symbol pairs on states

Note: because in FOL we only know total functions, in any *S*-structure $\mathcal{D}$ the function δ^A is totally defined. For the purposes of interpreting $\mathcal{D}$ as a DFA, it is not relevant which type of values δ^A has on argument pairs that are not of type (state, symbol).