

## Exercises for FLL, Fall 2018, sheet 12 - solutions

*For self-study purposes.*

**Exercise.** Using the rules from the sequent calculus, derive the rule

$$\frac{}{\Gamma \neg(\varphi \vee \neg\varphi) \zeta} \quad (\text{for any } \Gamma, \varphi, \zeta)$$

**Solution.** One possible derivation is:

- |                                             |                                  |                        |
|---------------------------------------------|----------------------------------|------------------------|
| (1) $\Gamma \neg(\varphi \vee \neg\varphi)$ | $\neg(\varphi \vee \neg\varphi)$ | (Pre)                  |
| (2) $\Gamma \varphi$                        | $\varphi$                        | (Pre)                  |
| (3) $\Gamma \neg\varphi$                    | $\neg\varphi$                    | (Pre)                  |
| (4) $\Gamma \varphi$                        | $(\varphi \vee \neg\varphi)$     | ( $\vee$ Con,a) on (2) |
| (5) $\Gamma \neg\varphi$                    | $(\varphi \vee \neg\varphi)$     | ( $\vee$ Con,b) on (3) |
| (6) $\Gamma$                                | $(\varphi \vee \neg\varphi)$     | (Cas) on (4),(5)       |
| (7) $\Gamma \neg(\varphi \vee \neg\varphi)$ | $(\varphi \vee \neg\varphi)$     | (Ant) on (6)           |
| (8) $\Gamma \neg(\varphi \vee \neg\varphi)$ | $\zeta$                          | (Con) on (1),(7)       |