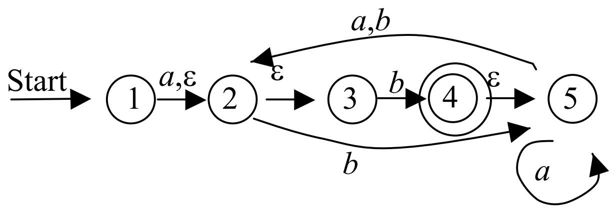
Exercises for FLL, Fall 2018, sheet 3

Return Thu Oct 4, in class. As always in this course you may work in teams of two if you wish – but not larger teams.

This is a two-week exercise sheet, twice the size as usual. I think it is wise to spread the work over two weeks.

Exercise 1. Design an ε-NFA that accepts the language denoted by $(((\varepsilon+a)bb)^*)a^*$. Represent your automaton by a transition diagram.

Exercise 2. Give a regular expression which describes the language accepted by the following ε-NFA:



Exercise 3. Is the language $L = \{w \in \{0, 1\}^* \mid w$ contains an equal number of 0's and 1's$\}$ regular? Prove your answer.

Exercise 4 [optional]. Prove that the language $L = \{0^n \mid n = pq$ for two primes $p, q\}$ is not regular.

Exercise 5. Prove the following claim:

Let M be some regular language over $\Sigma = \{0,1\}$. Define $L_{|M|} = \{0^n \in \{0\}^* \mid n = |v|$ for some word $v \in M\}$. Then L is regular.

Note: this is easy to solve using the tool of language homomorphisms – leads to a one-line proof. It is also possible to solve this problem without homomorphisms, also not difficult, though a bit more lengthy.

Exercise 6 A *sequence language* L over Σ is a language with two properties: (i) for each $n \geq 0$, there exists exactly one word in L of that length; (ii) if $u, v \in L$, $|u| < |v|$, then u is a prefix (= initial subword) of v.

- **a.** Give an example of a regular sequence language.
- **b.** Prove that every regular sequence language L is ultimately cyclic, that is, there exist words w and v such that L is the set of all initial substrings of the infinite sequence *wvvvv...* . *Hint*: you will benefit from the PL here.

Note. This was a midterm question in a long-time-ago FLL course.