Machine Learning, Spring 2018: Exercise Sheet 6

It's paper and pencil time again.

Problem 1 (linear algebra training). Let $x_1, ..., x_m \in \mathbb{R}^n$ be m linearly independent n-dimensional vectors, and let μ be their mean. Prove that the centered points $\bar{x}_1, \ldots, \bar{x}_m = x_1 - \mu, \ldots, x_m - \mu$ span an m–1 dimensional subspace of $\mathbb{R}^n$.

(Recall that a set $x_1, ..., x_m$ of vectors is called linearly independent if $a_1 x_1 + ... + a_m x_m = \mathbf{0}$ implies $a_1 = ... = a_m = 0$.)

Hint. Show two facts which combine into the desired claim. First, show that $x_1, ..., x_m$ are linearly dependent, that is they span a subspace of dimension less than m. Second, show that $x_1, ..., x_{m-1}$ are linearly independent, so these span a subspace of dimension $m - 1$.