

## PSM SPRING 2018, HOME STUDY MATERIAL 10

*Note. After the quiz 2 it appears advisable to train more about core probability concepts in a broad range. The following problems are thus scattered across themes from the entire course up to now, with no chronological order (because I wrote them as they came to my mind).*

1. Let  $(X_n)_{n=1,2,3,4,5}$  be a discrete-time stochastic process whose paths all have length 5. Let the  $X_n$  be i.i.d. with values in  $\{0,1\}$ , where  $P_{X_n}(\{0\}) = 0.1$ . Let  $Y_n$  (where  $n = 1, \dots, 5$ ) be defined by  $Y_n = X_1 + \dots + X_n$ . Your tasks: (a) Give the smallest possible sample space  $S_n$  for  $Y_n$ . (b) Draw a graphics that displays two realizations of the process  $(Y_n)_{n=1,2,3,4,5}$ .
2. Let  $\Omega = \{\omega_1, \dots, \omega_5\}$  be a (uncommonly small) universe. Define a probability measure  $P$  on  $\Omega$  and two RVs  $X, Y : \Omega \rightarrow \{\text{red, blue, green}\}$  which are identically distributed but not identical.
3. Let a three-state homogeneous Markov Chain with states  $\{a, b, c\}$  be given by the transition matrix  $M$  (whose rows sum to 1). Express the probabilities (a)  $P(X_2 = b \mid X_0 = a)$  and (b)  $P(X_2 = b \mid X_0 = a, X_1 = c)$  in terms of  $M$ .
4. A clinical survey on cancer patients reports that 95% of the patients with blood cancer had antibodies of a certain type A in their blood a year before the cancer became manifest, but from the members of a healthy control that was monitored for 1 year, only 10% carried the antibody A at the beginning of the observation year. Furthermore, it is since long known in the relevant medical literature that the chance for anybody to develop this cancer within 1 year is 0.01%. A doctor who know about all these findings sees a new patient Z of whom tests reveal that he carries antibody A.
  - (a) The clinical survey actually describes a statistical model with an underlying probability space  $(\Omega, \mathfrak{A}, P)$ , a sample space  $(E, \mathcal{F})$  and one or several random variables  $X$ . Give an informal description of  $\Omega$  (choose one possibility and detail it out), of  $E$  and the random variables  $X$ .
  - (b) What is the statistical chance of patient Z to develop blood cancer within one year?