PSM Spring 2019, homework 5

1. (The following probability numbers are my invention)

   - The probability that an adult person is suffering from schizophrenia is 0.01.
   - The probability that an adult person wearing a hearing aid hears voices is 0.1.
   - The probability that a schizophrenic adult person who has a hearing aid hears voices is 0.99.
   - The probability that an adult person has a hearing aid is independent of whether this person is schizophrenic.

   Mr. Thompson, who wears a hearing aid, hears voices. What is the probability that he suffers from schizophrenia?

2. Let $\Omega = \{\omega_1, \omega_2\}$ and $X, Y : \Omega \to \{0, 1\}$. Specify a probability measaure P on Ω and concrete values $X(\omega_i), Y(\omega_i)$ (where $i = 1, 2$) such that X, Y are identically distributed but not for all $\omega \in \Omega : X(\omega) = Y(\omega)$.

3. Verify the claim made in the last bullet point in the list after Definition 8.2.1 in the lecture notes.

4. Prove the equivalence of Definitions 12.0.1 and 12.0.2 in the lecture notes. *Note 1:* old version of the lecture notes had a wrong Definition 12.0.1. Use the definition given in the latest release (published online March 29). *Note 2:* This requires some work. My solution uses 1 full page of Latex-formatted formulas. But it all can be done with elementary transformations of conditional probabilities. If you master this one, you know how to play with those guys. If you don't find a way after some effort, check out the first half of the solution (which proves the first direction of the claimed equivalence). The "tricks" that you find there can be used also for the second direction - try your best. *Hint:* for both directions it is helpful to prove that a third equivalent definition of the Markov chain property follows from either 12.0.1 or 12.0.2, namely,

$$P(x_0, x_1, \ldots, x_n) = P(x_0)P(x_1 \mid x_0)P(x_2 \mid x_1) \cdots P(x_n \mid x_{n-1}).$$