Advanced Computer Science 1
First midterm, October 4, 2006

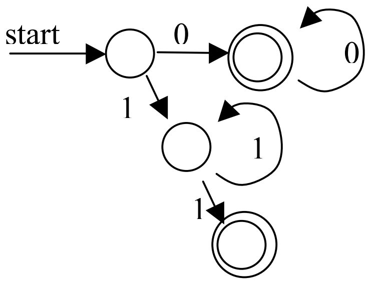
Solution sheet

Group A

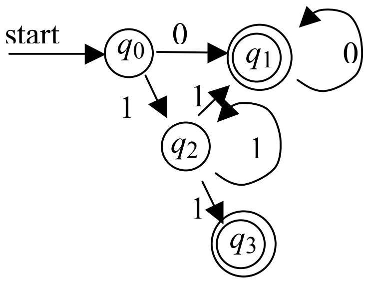
Note. You may quote any result from the lecture notes or a textbook within your solutions.
Note. A maximum of 100 points is accredited for this exam (sum of points of all problems = 110)

1. (10 points) Design an ε-NFA which accepts the language over $\Sigma = \{0,1\}$ that is described by the regular expression **00* + 11*1**. Present your ε-NFA by a transition diagram.

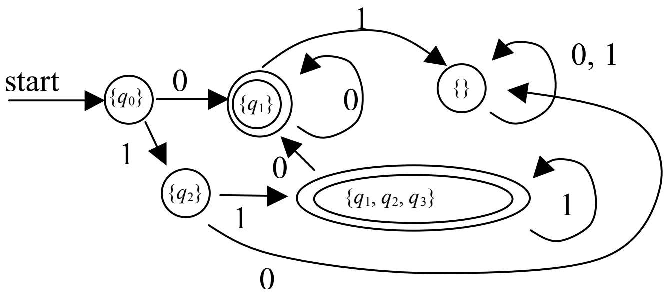
Solution.



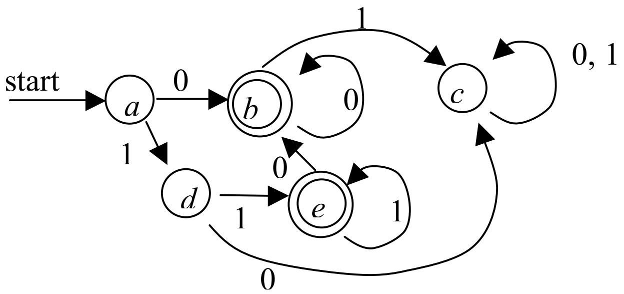
2. (15 points) Transform the following NFA into a DFA through the subset construction. Your DFA should have a totally defined transition function (that is, include a dead state if necessary). Present your DFA by a transition diagram, where the states are labelled by the sets of NFA states.



Solution.

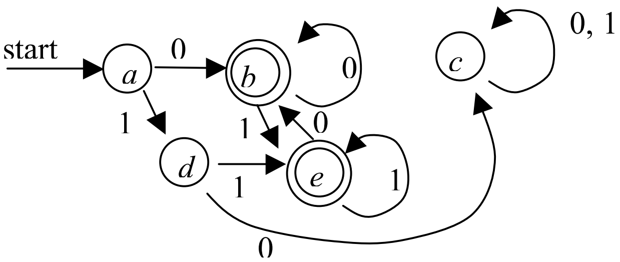


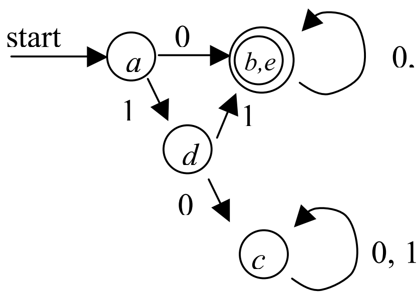
3. **a.** (15 points) Show that the DFA below is minimal. **b.** (15 points) Re-arrange a single transition arrow in the diagram below such that the resulting DFA is not minimal. Draw the re-arranged transition diagram and draw a transition diagram for the resulting minimal DFA.



Solution. a. Carrying out the table-filling algorithm, one finds that in the first round the state pairs (a, b), (a, e), (b, c), (b, d), (c, e), (d, e) are found to be distinguishable, and in the second round all remaining state pairs. Thus, no two states are indistinguishable and therefore the DFA is minimal.
b. One solution is the following:

re-arranged: minimalized:





4. (15 points) Show that the language $L = \{0^n1^{n^2} \in \{0,1\}^* \mid n \geq 0\}$ is not regular.

Solution. Pumping lemma! Assume L is regular with pumping constant k. Consider $w = 0^k1^{k2}$ $\in L$. By PL, $w = xyz$, with $|xy| \leq k$, $|y| > 0$. Because $|xy| \leq k$, y must consist entirely of 0's. By PL, then also $0^{k-|y|}1^{k2} \in L$, a contradiction.

5. (20 points) Let L be a regular language. Is the language $M = \{w^{|w|} \mid w \in L\}$ always regular? Prove your answer.

Solution. M is not always regular. For a counterexample, consider $L = L(\mathbf{0}^*)$. Then $M = \{0^{n^2} \mid n \geq 0\}$. A pumping lemma argument shows that M is not regular. (This argument should be carried out in the exam solution; or it might be quoted from a textbook).

6. (20 points) Let L be a regular language, and K be a finite language. Show that $M = \{w^k \mid w \in K, \text{ there exists } v \in L \text{ with } |v| = k\}$ is regular.

Solution. Let $K = \{w_1, ..., w_n\}$. Then $M = \{w_1^{|v|} \mid v \in L\} \cup ... \cup \{w_n^{|v|} \mid v \in L\}$. Since the regular languages are closed under finite unions, it suffices to show that $\{w_1^{|v|} \mid v \in L\}$ is regular. Let Σ_K and Σ_L be the alphabets of K and L, respectively. Define a homomorphism h: $\Sigma_L \rightarrow \Sigma_K{}^*$ by $h(a) = w_1$ for all $a \in \Sigma_L$. Then obviously $\{w_1^{|v|} \mid v \in L\} = h(L)$, hence this language is regular.

Advanced Computer Science 1
Second midterm, November 8, 2006

Group A

Solution sheet

Note. A maximum of 100 points is accredited for this exam (sum of points of all problems = 110)

Problem 1 A *sequence language* L over Σ is a language with two properties: (i) for each $n \geq 0$, there exists exactly one word in L of that length; (ii) if $u, v \in L$, $|u| < |v|$, then u is a prefix (= initial subword) of v.

- **a.** (5 points, 5 minutes) Give an example of a regular sequence language.
- **b.** (25 points, 15 minutes) Prove that every regular sequence language L is ultimately cyclic, that is, there exist words w and v such that L is the set of all initial substrings of the infinite sequence *wvvvv...* . *Hint*: you will benefit from the PL here.

Solution. a. The simplest example is surely the language $L(\mathbf{1}^*)$.

b. Let L be a regular sequence language. Because L is regular, the PL holds for L. Let n be a PL constant for L. Because a sequence language is infinite, there must exist some w in L with $|w| \geq n$. By the PL, w can be split into $w = xyz$, where $|y| > 0$, $|xy| \leq n$, and every xy^kz (where $k \geq 0$) is in L. Thus L contains all the words xz, xyz, $xyyz$, $xyyyz$, etc. Because in a sequence language L it holds that if $u \in L$, $v \in L$, $|v| < |u|$, then $u = vx$ for some x, we can conclude from $xy^kz \in L$ that $x, xy, xyy, ..., xy^k \in L$. Because $xy^kz \in L$ for all k, we conclude that $xy^k \in L$ for all k. By the fact that sequence languages are closed under initial subwords, it follows that all initial strings of the infinite sequence *xyyy...* are in L. Because L is a sequence language, no other words can be in L. Thus L is the language of all initial substrings of *xyyy...*, hence L is ultimately cyclic.

Problem 2 (40 points, 30 minutes) A "yes-PDA" $P_{\text{yes}} = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$ is defined like ordinary PDAs, with the extra condition that $\{Y, E, S\} \subseteq \Gamma$. Then the *language accepted by* P_{yes} *by a yes-answer* is

$$L(P_{\text{yes}}) = \{w \in \Sigma^* \mid \text{there exists } q \in Q \text{ such that } (q_0, w, Z_0) \vdash_{P_{\text{yes}}}^* (q, \varepsilon, YES)\}.$$

Show that the languages that can be accepted by some yes-PDA by yes-answers are context-free. (Note: the B group of this exam have to show the converse statement that the context-free languages are accepted by yes-PDAs by yes-answers – so in fact, the "yes-PDA-acceptable" languages are exactly the context-free languages.)

Solution. Let $\mathbf{L}_{\text{yes}}$ be the set of all languages that can be accepted by some yes-PDA by a yes-answer, and $\mathbf{L}_{\text{cf}}$ the set of all context-free languages. We show $\mathbf{L}_{\text{yes}} \subseteq \mathbf{L}_{\text{cf}}$. Let $L \in \mathbf{L}_{\text{yes}}$, and let $P_{\text{yes}} = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$ accept L by a yes-answer. We construct a PDA $P = (Q', \Sigma, \Gamma', \delta', q_0', Z_0, \{u\})$ that accepts L by final state u.

We put

$Q' = Q \cup \{q_0', r, s, t, u\}$ [all are new states not already in Q]

$\Gamma' = \Gamma \cup \{Z_1\}$ [Z_1 not in Γ]

δ' is δ plus the following new transitions. We first add one transition which at startup replaces Z_0 by Z_0Z_1 and goes to the old start state q_0:

- $(q_0', \varepsilon, Z_0) \to (q_0, Z_0Z_1)$

Now the old rules from δ allow P on input w to reach a configuration $(q, \varepsilon, YESZ_1)$ iff w is in the language of P_{yes}.

We further add to δ the following rules that can empty away a top *YES* from the stack without processing input:

- For all $q \in Q$, add the rule $(q, \varepsilon, Y) \to (r, \varepsilon)$ [idea: start deleting *YES*]
- In addition, add the rules $(r, \varepsilon, E) \to (s, \varepsilon)$, $(s, \varepsilon, S) \to (t, \varepsilon)$ [idea: complete deletion of *YES*]
- Finally, add the rule $(t, \varepsilon, Z_1) \to (u, \varepsilon)$.

Clearly this will bring P to its accepting state u iff w is in the language of P_{yes}.

Problem 3. Consider the language $L = \{10^k1^k0 \mid k \geq 0\}$.

a. (20 points, 10 minutes) Give a CNF grammar for this language.

b. (5 points, 3 minutes) Show that there exists no grammar for L whose productions are all of the type $A \to BCD$ or $A \to a$ (where A, B, C, D are variables and a is a terminal).

Solution. a. An obvious grammar for L is $S \to 10 \mid 1R0$, $R \to 01 \mid 0R1$. Transforming this into CNF is easy because this grammar already has no ε-productions, no unit pairs, and all symbols are useful. Thus only the last steps in the construction of CNFs are needed. The first production $S \to 10$ yields new rules $S \to AB$, $A \to 1$, $B \to 0$; the second production $S \to 1R0$ yields new rules $A_1 \to 1$, $A_0 \to 0$, $S \to A_1T$, $T \to RA_0$; the production $R \to 01$ is replaced by $R \to A_0 A_1$, and the production $R \to 0R1$ gives $R \to A_0U$, $U \to RA_1$.

b. The word 10 cannot be generated with rules of the kind $A \to BCD$ or $A \to a$, because (i) it clearly cannot be generated with rules of the kind $A \to a$ only, (ii) thus at least one call of a rule of the type $A \to BCD$ would be needed in any derivation of 10, (iii) any word w of terminals resulting from a derivation that calls some rule of the kind $A \to BCD$ would result in $|w| \geq 3$, because of the absence of ε-productions.

Problem 4 (10 points, 8 minutes) Show that the word *ababa* is in the language of the CNF grammar with the productions $S \to a \mid AB \mid AA$; $A \to a \mid BA$; $B \to b \mid SS$ (S is the start symbol).

Solution. Applying the CYK algorithm yields the table

$\{B\ S\ A\}$				
$\{B\}$	$\{S\ B\}$			
$\{S\ B\}$	$\{S\}$	$\{S\ B\}$		
$\{S\}$	$\{A\}$	$\{S\}$	$\{A\}$	
$\{S\ A\}$	$\{B\}$	$\{S\ A\}$	$\{B\}$	$\{S\ A\}$
a	*b*	*a*	*b*	*a*

Since S figures in the top cell, S generates our target word *ababa.*

Problem 5 (5 points, 2 minutes). Give a complete list of all terms that are contained in the FOL expression

$$\forall x\ \forall y\ ((\mathsf{Vec}\ x \wedge \mathsf{Vec}\ y) \rightarrow \forall r\ \forall s\ ((\mathsf{Scal}\ r \wedge \mathsf{Scal}\ s)\cdot \rightarrow \mathsf{Vec} + \cdot\ r\ x \cdot s\ y))),$$

where Vec and Scal are unary predicate symbols and + and · are binary function symbols (you might read this expression as "the sum of two scalar-weighted vectors is a vector").

Solution. The terms are x, y, r, s, $\cdot\ r\ x$, $\cdot\ s\ y$, and $+ \cdot\ r\ x \cdot s\ y$.

Advanced Computer Science 1
Final exam, December 15, 2006

Group A

Solution sheet

Problem 1 (8 points) Here are four languages over $\Sigma = \{0, 1\}$:

$L_1 = \{0^{10n}1^{20k} \mid n, k > 0\}$ $L_2 = \{0^{10n}1^{20n} \mid n > 0\}$

$L_3 = \{0^n1^k \mid n > k\}$ $L_4 = \{10^n1 \mid n > 1000\}$

Which of the following statements are correct? Please mark on your solution sheet.

(a) L_1 and L_3 are regular N
(b) L_2 and L_4 are regular N
(c) Exactly two among these four languages are regular Y
(d) All four languages are context-free Y

(a) L_1 and L_2 are regular N
(b) L_3 and L_4 are regular N
(c) Exactly three among these four languages are regular N
(d) All four languages are context-free Y

(a) L_1 and L_2 are regular N
(b) L_3 and L_4 are regular n
(c) Exactly two among these four languages are regular Y
(d) All four languages are context-free Y

(a) L_1 and L_3 are regular N
(b) L_2 and L_4 are regular N
(c) Exactly three among these four languages are regular N
(d) All four languages are context-free Y

Key: 2 points per correct answer, –2 points per incorrect answer, 0 points for no answer.

Problem 2 (16 points) Which of the following statements is correct? Mark the correct ones on the solution sheet!

(a) Given a regular language L, the minimal DFA accepting L has always exactly as many states as any minimal-size NFA accepting L. N

(b) If A is a DFA for a language L, and A has two states $q \neq p$ such that for some nonempty word w it holds that $\delta(q, w) = \delta(p, w)$ and $\delta(p, w)$ is an accepting state, then A is not minimal. N

(c) An NFA for a language L can be transformed into a minimal DFA for L by first carrying out the subset construction and then deleting all inaccessible states. N

(d) Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Define for every $q \in Q$ the language $L(q) = \{w \in \Sigma^* \mid \delta(q, w) \in F\}$. If for all $p, q \in Q$, $q \neq p$, it holds that $L(p) \cap L(q) = \emptyset$, then A is minimal. Y

(a) Given a regular language L, the minimal DFA accepting L may have more states than a minimal-size NFA accepting L. Y

(b) If $A = (Q, \Sigma, \delta, q_0, F)$ is a DFA for a language L, and A has two states $q \neq p$ such that for every word w it holds that $\delta(q, w) \in F \Leftrightarrow \delta(p, w) \in F$, then A is not minimal. Y

(c) The subset construction transforms a given NFA into an equivalent DFA, which however need not be minimal. To make it minimal, in a final step the inaccessible states have to be detected and removed. N

(d) Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Define for every $q \in Q$ the language $L(q) = \{w \in \Sigma^* \mid \delta(q, w) \in F\}$. If for all $p, q \in Q$, $q \neq p$, it holds that $L(p) \cap L(q) = \emptyset$, then A is minimal. Y

(e) Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Define for every $q \in Q$ the language $L(q) = \{w \in \Sigma^* \mid \delta(q, w) \in F\}$. If for all $p, q \in Q$, $q \neq p$, it holds that $L(p) \cap L(q) = \emptyset$, then A is minimal. Y

(a) Given a regular language L, the minimal DFA accepting L has always exactly as many states as any minimal-size NFA accepting L. N

(b) If $A = (Q, \Sigma, \delta, q_0, F)$ is a DFA for a language L, and A has two states $q \neq p$ such that for every word w it holds that $\delta(q, w) \in F \Leftrightarrow \delta(p, w) \in F$, then A is not minimal. Y

(c) The subset construction transforms a given NFA into an equivalent DFA, which however need not be minimal. To make it minimal, in a final step the inaccessible states have to be detected and removed. N

(a) An NFA for a language L can be transformed into a minimal DFA for L by first carrying out the subset construction and then deleting all inaccessible states. N

(b) Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Define for every $q \in Q$ the language $L(q) = \{w \in \Sigma^* \mid \delta(q, w) \in F\}$. If for all $p, q \in Q$, $q \neq p$, it holds that $L(p) \cap L(q) = \emptyset$, then A is minimal. Y

(c) If A is a DFA for a language L, and A has two states $q \neq p$ such that for some nonempty word w it holds that $\delta(q, w) = \delta(p, w)$ and $\delta(p, w)$ is an accepting state, then A is not minimal. N

(d) Given a regular language L, the minimal DFA accepting L may have more states than a minimal-size NFA accepting L. Y

Key: 4 points per correct answer, –4 points per incorrect answer, 0 points for no answer.

Problem 3 (12 points) Here are four regular expressions over the alphabet $\{a, b\}$:

$E_1 = (\mathbf{ab} + \mathbf{a}^*\mathbf{b}^*\mathbf{b}^*)^*$ $E_2 = ((\mathbf{ab})^* (\mathbf{a}^*\mathbf{b}^*\mathbf{b}^*)^*)^*$ $E_3 = (\mathbf{a} + \mathbf{b})^*$ $E_4 = \mathbf{a}(\mathbf{a} + \mathbf{b})^*$

Which of the following statements are true? Mark them in your solution sheet.

(a) $L(E_2) = L(E_3)$ Y

(b) $L(E_3) = L(E_4)$ N

(c) $L(E_1) = L(E_4)$ N

(d) The minimal DFA for $L(E_1)$ has five states. N

(e) The minimal DFA for $L(E_3)$ has two states. N

(f) The minimal DFA for $L(E_4)$ has two states. Y

(a) $L(E_1) = L(E_3)$ Y

(b) $L(E_3) = L(E_4)$ N

(c) $L(E_2) = L(E_4)$ N

(d) The minimal DFA for $L(E_2)$ has five states. N

(e) The minimal DFA for $L(E_3)$ has one state. Y

(f) The minimal DFA for $L(E_4)$ has one state. N

(a) $L(E_1) = L(E_3)$ Y

(b) $L(E_3) = L(E_4)$ N

(c) $L(E_2) = L(E_4)$ N

(d) The minimal DFA for $L(E_2)$ has five states. N

(e) The minimal DFA for $L(E_3)$ has one state. Y

(f) The minimal DFA for $L(E_4)$ has one state. N

(a) $L(E_1) = L(E_3)$ Y

(b) $L(E_3) = L(E_4)$ N

(c) $L(E_2) = L(E_4)$ N

(d) The minimal DFA for $L(E_1)$ has five states. N

(e) The minimal DFA for $L(E_3)$ has two states. N

(f) The minimal DFA for $L(E_4)$ has two states. Y

Key: 2 points per correct answer, –2 points per incorrect answer, 0 points for no answer.

Problem 4 (20 points) A grammar $G = (V, T, P, S)$ is called *reduced* if for every proper subset $P' \subset P$, which results in a grammar $G' = (V, T, P', S)$, the language $L(G')$ is a proper sublanguage of $L(G)$. Which of the following statements are correct? Mark in your solution sheet!

(a) Every grammar in Chomsky normal form is reduced. N

(b) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a grammar for L in Chomsky normal form which has at least n variables. Y

(c) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a reduced grammar for L in Chomsky normal form which has at least n variables. Y

(d) If a grammar is reduced, it does not contain unit productions. N

(a) If a grammar is reduced, all its variables are generating. Y

(b) Every grammar in Chomsky normal form is reduced. N

(c) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a grammar for L in Chomsky normal form which has at least n variables. Y

(d) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a reduced grammar for L in Chomsky normal form which has at least n variables. Y

(a) Every grammar in Chomsky normal form is reduced. N

(b) If a grammar is reduced, all its variables are generating. Y

(c) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a grammar for L in Chomsky normal form which has at least n variables. Y

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(a) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a grammar for L in Chomsky normal form which has at least n variables. Y

(b) For every infinite context-free language L which does not contain ε, and every $n \in \mathbb{N}$, there exists a reduced grammar for L in Chomsky normal form which has at least n variables. Y

(c) Every grammar in Chomsky normal form is reduced. N

(d) If a grammar is reduced, it does not contain unit productions. N

Key: 5 points per correct answer, –5 points per incorrect answer, 0 points for no answer.

Problem 5 (8 points). Here are three natural-language sentences S1 – S3, and four FOL expressions $\varphi_1 - \varphi_4$:

S1: Every man loves a woman. S2: All men love all women.

S3: Some man loves a woman.

φ_1: $\forall x\ \exists y$ (Man x ∧ (Woman y ∧ Loves x y)) φ_2: $\forall x\ \exists y$ (Man x → (Woman y ∧ Loves x y))

φ_3: $\forall x\ \forall y$ (Man x ∧ (Woman y ∧ Loves x y)) φ_4: $\exists x\ \exists y$ (Man x ∧ (Woman y ∧ Loves x y))

Please mark on the solution sheet all of the following statements which you judge correct.

(a) φ_1 is a correct formalization of S1. N

(b) φ_2 is a correct formalization of S1. Y

(c) φ_3 is a correct formalization of S2. N

(d) φ_4 is a correct formalization of S3. Y

(a) φ_1 is a correct formalization of S1. N

(b) φ_3 is a correct formalization of S2. N

(c) φ_2 is a correct formalization of S1. Y

(e) φ_4 is a correct formalization of S3. Y

(a) φ_2 is a correct formalization of S1. Y

(b) φ_1 is a correct formalization of S1. N

(c) φ_4 is a correct formalization of S3. Y

(d) φ_3 is a correct formalization of S2. N

(a) φ_2 is a correct formalization of S1. Y

(b) φ_4 is a correct formalization of S3. Y

(c) φ_1 is a correct formalization of S1. N

(d) φ_3 is a correct formalization of S2. N

Key: 2 points per correct answer, –2 points per incorrect answer, 0 points for no answer.

Problem 6 (16 points). Here are the axioms of group theory:

φ_1: $\forall x \forall y \forall z \ (x \circ y) \circ z = x \circ (y \circ z)$

φ_2: $\forall x \ x \circ e = x$

φ_3: $\forall x \exists y \ x \circ y = e$

The signature is $S = \{e, \circ\}$, where e is a constant symbol and $\circ$ is a binary function. Here are two S-structures:

$\mathcal{A} = (A, e^{\mathcal{A}}, \circ^{\mathcal{A}})$, where $A = \{1, 2\}$, $e^{\mathcal{A}} = 1$ and $\circ^{\mathcal{A}}$ is given by the table below;

$\mathcal{B} = (B, e^{\mathcal{B}}, \circ^{\mathcal{B}})$, where $B = \{1, 2, 3\}$, $e^{\mathcal{B}} = 1$ and $\circ^{\mathcal{B}}$ is given by the table below.

$\circ^{\mathcal{A}} =$

	2	2
1	1	1
2	1	1

$\circ^{\mathcal{B}} =$

	1	2	3
1	1	1	1
2	2	2	2
3	3	3	3

In the solution sheet, please tick the statements which are correct.

(a) φ_1 is satisfiable Y

(b) $\mathcal{A} \models \varphi_3$ Y

(c) $\varphi_1 \models \varphi_2$ N

(d) $\mathcal{B} \models (\varphi_3 \rightarrow \varphi_1)$ Y

(a) φ_2 is satisfiable Y

(b) $\mathcal{B} \models \varphi_1$ Y

(c) $\varphi_3 \models \varphi_2$ N

(d) $\mathcal{A} \models (\varphi_3 \rightarrow \varphi_1)$ Y

(a) φ_3 is satisfiable Y

(b) $\mathcal{A} \models \varphi_2$ N

(c) $\varphi_1 \models \varphi_2$ N

(d) $\mathcal{B} \models (\varphi_2 \rightarrow \varphi_1)$ Y

(a) φ_2 is satisfiable Y

(b) $\mathcal{B} \models \varphi_3$ N

(c) $\varphi_2 \models \varphi_3$ N

(d) $\mathcal{A} \models (\varphi_3 \rightarrow \varphi_1)$ Y

Key: 4 points per correct answer, –4 points per incorrect answer, 0 points for no answer.

Problem 7. (20 points) Which of the following statements is correct? Mark the correct ones on the solution sheet!

(a) It is not decidable whether $\Phi \vdash \varphi$. Y

(b) If Φ is consistent and Ψ is inconsistent, then $\Psi \cup \Phi$ may or may not be consistent, depending on Ψ and Φ. N

(c) If Φ is consistent and Ψ is inconsistent, then $\Psi \cap \Phi$ is consistent. Y

(d) If $\Phi \cup \{ \neg\varphi\}$ is not satisfiable, then $\Phi \cup \{\varphi\}$ is satisfiable. N

(e) Let the signature *S* be finite. Then the set $\{ \varphi \mid \emptyset \models \varphi \}$ of tautological *S*-expressions is finite. N

(a) It is decidable whether $\Phi \vdash \varphi$. N

(b) If Φ is consistent and Ψ is inconsistent, then $\Psi \cap \Phi$ may or may not be consistent, depending on Ψ and Φ. N

(c) If Φ is consistent and Ψ is inconsistent, then $\Psi \cup \Phi$ is consistent. N

(d) If $\Phi \cup \{\varphi\}$ is satisfiable, then $\Phi \cup \{\neg\varphi\}$ is not satisfiable. N

(e) Let the signature *S* be finite. Then the set $\{\varphi \mid \varphi \models (\psi \wedge \neg\psi)\}$ of contradictory *S*-expressions is finite. N

(a) Let the signature *S* be finite. Then the set $\{\varphi \mid \varphi \models \psi \wedge \neg\psi \}$ of contradictory *S*-expressions is finite. N

(b) If $\Phi \cup \{\varphi\}$ is satisfiable, then $\Phi \cup \{\neg\varphi\}$ is not satisfiable. N

(c) If Φ is consistent and Ψ is inconsistent, then $\Psi \cap \Phi$ may or may not be consistent, depending on Ψ and Φ. N

(d) It is not decidable whether $\Phi \vdash \varphi$. Y

(e) If Φ is consistent and Ψ is inconsistent, then $\Psi \cap \Phi$ is consistent. Y

(a) If Φ is consistent and Ψ is inconsistent, then $\Psi \cup \Phi$ may or may not be consistent, depending on Ψ and Φ. N

(b) Let the signature S be finite. Then the set $\{ \varphi \mid \varnothing \vDash \varphi \}$ of tautological S-expressions is finite. N

(c) If $\Phi \cup \{ \neg\varphi\}$ is not satisfiable, then $\Phi \cup \{\varphi\}$ is satisfiable. N

(d) If Φ is consistent and Ψ is inconsistent, then $\Psi \cup \Phi$ is consistent. N

(e) It is decidable whether $\Phi \vdash \varphi$. N

Key: 4 points per correct answer, –4 points per incorrect answer, 0 points for no answer.