FLL, Fall 2010: Final Exam

Solutions

Each of the following statements is either true or false. Fill the corresponding answer box with a "T" if you think the statement is true, and with a "F" if you think it is false. If you change your mind and want to flip your first mark, cross it out and write your final judgement besides it in an unambiguous manner. Unfilled or ambiguously marked boxes will be considered as wrong answers.

Exam point calculation: There are altogether 30 questions. If you get all of them right, the exam score will be 100. If you get 15 of them right (which amounts to random guesses), the exam score will be 40 points, corresponding to a Jacobs grade of 5.0. If you get N questions right, linear interpolation will be used to obtain the exam points, i.e. the formula exam_points =4 $N - 20$ *will be used. Plus, there is a bonus question at the end of this sheet.*

Answer box	Statement
T	1. There are as many regular languages as there are context-free languages (over a given alphabet)
T	2. If L is a regular language, and M is a finite language, then $L \cap M$ is regular.
T	3. If L is a regular language over alphabet Σ, then $M = \{w \in \Sigma^* \mid w^r \notin L\}$ is regular (where w^r is the reverse of w)
T	4. If L is a regular language whose minimal DFA has k states, then there exists a context-free grammar for L with k variables.
F	5. If L is a regular language whose minimal DFA has k states, then any NFA for L has at least k states.
F	6. If $L_1, L_2, L_3,...$ is a sequence of regular languages over the same alphabet, then the language $M = \{w_i \mid w_i$ is the first word in the alphabetical enumeration of $L_i\}$ is context-free.
T	7. There exist regular languages L, M, and a homomorphism h with $M = h(L)$, where L is infinite and M is finite
T	8. If L is regular, then there exists a deterministic pushdown automaton that accepts L.
F	9. The pumping lemma can be used as a decision algorithm to decide whether a given language is regular or not.
F	10. If L is some language, then the union of all regular sublanguages of L is regular.
F	11. The following equivalence statement holds for any two regular expressions with language variables: $(L_1 + L_2)^* =_\Sigma L_1^* + L_1^*$
F	12. Let h be the homomorphism $h(a) = 11$, $h(b) = 00$, and let $L = (101)^*$. Then $h^{-1}(L)$ is not defined.

Answer box	Statement
T	**13.** Let π: $V \to V$ be a permutation on the variable set of a CFG A which leaves the start variable invariant. Let B be the grammar obtained by replacing in all rules of A the variables with their images under π. Then $L(A)$ is finite iff $L(B)$ is finite.
T	14. Let $L = \{w\}$. Then the equivalence relation R_L from the Myhill-Nerode theorem has $\|w\| + 2$ equivalence classes.
T	15. There exists an algorithm which, when given as input an integer n and a CFG A, outputs the n-th word of the alphabetical enumeration of $L(A)$ provided that $\| L(A) \| \geq n$, and "fail" if $\| L(A) \| < n$.
F	16. The language of the following CFG is infinite: $S \to AB \mid 0$ $A \to 01 \mid 1 \mid A \mid B$ $B \to 0$
T	17. If L is a context-free language over Σ, and $w \in \Sigma^*$ a word, then the language $\{xw \in \Sigma^* \mid x \in L\}$ is context-free.
T	18. For any regular language L, there exist countably infinite many non-isomorphic context-free grammars (two grammars A and B are isomorphic if B can be obtained from A by a renaming of variables)
T	19. It is decidable whether a pushdown automaton accepts the empty word.
F	20. It is decidable whether two pushdown automata accept the same language.
T	21. Regular languages always admit unambiguous grammars.
F	22. Let $S = \{c, R, f\}$ be a signature where c is a constant symbol, R a ternary relation symbol, and f a unary function symbol. Then, $fRccc = Rfccc$ is a well-formed S-expression.
T	23. Let R be a binary relation symbol. Then $\neg (((Rx_1x_1 \vee Rx_1x_2) \vee Rx_2x_1) \vee Rx_2x_2)$ is satisfiable.
F	24. Consider a signature consisting of a single constant symbol c, and let $\Phi = \{\neg x_i = c \mid i \in \mathbb{N}\}$, i.e. Φ contains one formula of the kind $\neg x = c$ for every variable x. Then Φ has no countable model.
Answer box	**Statement**
T	25. For the empty signature $S = \varnothing$, there exists an S-expression φ without free variables, such that, if $\mathcal{A} \models \varphi$, then the carrier of $\mathcal{A}$ has exactly three elements.
F	26. If an S-expression φ has no free variables and is neither a tautology nor a contradiction, it has (up to isomorphism) exactly one model.
T	27. If $\neg(\neg\psi \vee \varphi)$ is unsatisfiable, then $\psi \vdash \varphi$.
F	28. For a finite signature S, there exist only finitely many non-isomorphic S-structures.
T	29. If the sequent $\psi\ \varphi$ can be derived in the sequent calculus, then $(\psi \to \varphi)$ is a tautology.

F	30. It is decidable whether for two *S*-expressions ψ and φ it holds that $\psi \models \varphi$.

Optional bonus question (estimated processing time: 20 minutes) *For this bonus question, a maximum of 10 points is added to the point score obtained from the multiple-choice part of this exam. If in this way you reach an exam score above 100, it will be counted toward the course grade.*

If you use separate sheets for the bonus question, make sure your name is indicated on each sheet.

Show that the regular languages are closed under 1-shift. The 1-shift of a language *L* over an alphabet Σ is the language

$$L_{1shift} = \{w \in \Sigma^* \mid w = va,\ a \in \Sigma, \text{ and } av \in L; \text{ or } w = \varepsilon, \text{ and } \varepsilon \in L\}.$$